$$\frac{1}{2}mv^2 = h\nu - eV_0 = eV_s \tag{4.28}$$

which is identical to Eq. (4.26), with $k = h/e$. The photoelectric effect provides another means to determine Planck's constant h originally used by Planck for blackbody radiation and by Bohr for the hydrogen spectrum.

Furthermore, since the energy of each photon is $h\nu$, the intensity of the radiation is not related to the energy of each photon, but instead determines the number of photons striking the metal surface per second. The rate of electron ejection is expected to be proportional to the rate at which the photons impinge upon the metal surface; thus, an increase in light intensity is predicted to increase the photoelectric current, as observed. Because the amount of energy absorbed by an electron is $h\nu$ regardless of the rate at which photons impinge on the surface, the kinetic energy of the ejected electrons should be independent of the intensity of the light. Thus, all of the predictions of the photon mechanism for the photoelectric effect are in agreement with the experimental results.

COMPTON EFFECT

An experiment that is related to the photoelectric effect is the Compton effect. This experiment, which provides more detailed information about the interaction of radiation and matter was performed in the early 1920's and analyzed by Compton in 1923. The experiment comprises the irradiation of a sample of material such as a paraffin hydrocarbon with X-rays or γ-rays, high-frequency radiation. The photons are scattered from bound electrons, which are ionized. The wavelength of the scattered radiation and the energy of the emitted electron are determined as a function of angle, relative to the incident beam. It is found that the radiation scattered from the material contains not only wavelengths equal to that of the incident radiation λ, but also wavelengths of the order of a few hundredths of an Angstrom longer than λ. The dependence of the scattered wavelength λ' upon the angle θ between the primary and scattered beams is found to be:

$$\lambda' = \lambda + k \sin^2\left(\frac{\theta}{2}\right) \tag{4.29}$$

where k is a constant.

Physicists of the early 20th century had a misconception regarding classical wave theory and the Compton effect that has been promulgated to the present. They erroneously predicted that the wavelength of the radiation would increase based on the Doppler effect since an electron in the sample would be accelerated by the impinging radiation and would therefore emit waves with longer wavelengths. The Doppler effect does not correctly explain the observations, however, since (a) the Doppler shift is proportional to the wavelength of the primary radiation and (b) the Doppler shift increases with the electron velocity and therefore should increase with time, since the electrons are accelerated continuously while they absorb energy during the irradiation. Neither of these predictions is corroborated by the experimental results, not as a consequence of the failure of classical theory, but because of an erroneous misconception about the nature of the photon and its interaction with matter. As was the case for the photoelectric effect, the observations can be explained quantitatively by the photon theory of radiation given *supra* and the laws of conservation of energy and momentum for particles including photons and electrons.

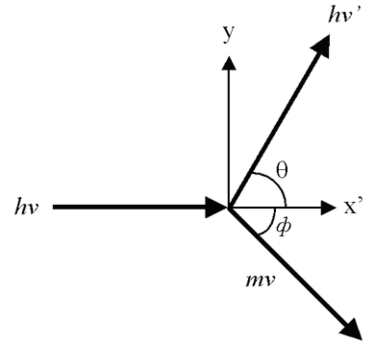
According to Eqs. (2.148-2.150), the incident photon with wavelength λ and frequency $\nu = c/\lambda$ has a momentum $h\nu/c$. Correspondingly, the scattered photon, which has a longer wavelength λ', and therefore a lower frequency $\nu' = c'/\lambda'$, has a lower momentum $h\nu'/c$. Since ν is in the X-ray region $(\lambda \sim 1-10\ \text{Å})$, the energy $(h\nu \sim 1000\ \text{eV})$ is so much greater than the binding energy of the electrons $(\approx 10\ \text{eV})$ that to a first approximation the latter be neglected. Thus, the electron is ejected in the direction ϕ with a momentum mv, which is calculable from an energy and momentum balance for the process as shown in Figure 4.10. The classical equations of conservation of energy and of the two components of the linear momentum are:

$$h\nu = h\nu' + \frac{1}{2}m_e v^2 \qquad \text{(energy)} \tag{4.30}$$

$$\frac{h\nu}{c} = \frac{h\nu'}{c}\cos\theta + m_e v\cos\phi \qquad (x \text{ component of momentum}) \tag{4.31}$$

$$0 = \frac{h\nu'}{c}\sin\theta - m_e v\sin\phi \qquad (y \text{ component of momentum}) \tag{4.32}$$

Figure 4.10. The Compton effect based on conservation of energy and momentum of a scattered photon and an electron.



Eliminating v and ϕ from these equations, introducing λ by the definition $\lambda = c/v$, and making the approximation that $\lambda\lambda' \approx \lambda^2$, gives:

$$\Delta\lambda = \lambda' - \lambda = 2\frac{h}{m_e c}\sin^2\left(\frac{\theta}{2}\right) \tag{4.33}$$

in agreement with Eq. (4.29). For λ in Angstroms, Eq. (4.33) gives:

$$\Delta\lambda = 0.0485\ \sin^2\left(\frac{\theta}{2}\right) \tag{4.34}$$

If the ejected electron is treated relativistically with its total energy given by Eq. (34.17):

$$E = \left(m_e^2 c^4 + p_e^2 c^2\right)^{1/2} = m_e c^2\sqrt{1+\left(\frac{v_e}{c}\right)^2} \tag{4.35}$$

and the kinetic energy is obtained by subtracting the rest energy $m_e c^2$, Eq. (4.33) can be derived without using the approximation that $\lambda' \approx \lambda$. The maximum shift is seen to occur for $\theta = \pi$, where $\Delta\lambda$ = 0.0485 Å.

The photon mechanism was tested by using γ-rays of energy $\approx 10^6$ eV, and the scattered photon and the Compton electron were recorded by means of scintillation counters. Cross and Ramsey [18] found that the angles ϕ and θ for an electron and a photon which were simultaneously detected were within $\pm 1^\circ$ of those required by the conservation laws (Eqs. 4.30-4.32)).

The analysis of the photoelectric and Compton effects shows that the particle viewpoint and Newtonian mechanics lead to a simple and quantitatively correct interpretation of these experiments, and that predictions based upon the classical wave theory are *not* wrong, but must be understood from the nature of the photon given by Eqs. (4.4-4.7). Individual photons behave as particles with energy given by Planck's equation (Eq. (4.8)). As shown by Eqs. (4.18-4.23), photons superimpose to give a spherical wave which gives rise to certain other phenomena such as diffraction and interference which are typically ascribed to wave theory with waves as an independent aspect of photons. The character exhibited by radiation, whether wave-like or particle-like, depends upon the type of experiment that is done. If the interaction of radiation with matter produces a measurable change in the matter, such as the ejection of an electron, the phenomenon appears to require the photon theory for its interpretation. If the interaction produces a measurable change in the spatial distribution of the radiation, such as diffraction at a slit, but produces no measurable change in the matter, invoking the wave theory seems appropriate as shown in the Classical Scattering of Electromagnetic Radiation section. Superficially, these results suggest that a synthesis of the two points of view is required which takes into account the nature of the experiment being analyzed; that is, the measuring process itself must be included in the theory. In actuality, both particle and wave aspects arise naturally from the particle-like photons which superimpose in time or space to form a wave which accounts precisely for the wave-particle duality of light.

TRANSITIONS

Other interactions involving electromagnetic radiation and matter are given classically wherein the photon carries $\hbar$ of angular momentum in its electric and magnetic fields as given by Eq. (4.1) with a corresponding energy given by Planck's equation (Eq. (4.8)). Bremsstrahlung radiation is given classically as radiation due to acceleration of charged particles by Jackson [19]. *Cherenkov radiation* occurs when charges moving at constant velocity in a medium different from vacuum possess spacetime Fourier components of the current that are synchronous with a wave traveling at the speed of light as given by a radiative condition derived from Maxwell's equations by Haus [20]. That is spacetime harmonics of $\frac{\omega_n}{c}\sqrt{\frac{\varepsilon}{\varepsilon_0}} = k$ do exist for which the Fourier transform of the current-density function is nonzero [20].