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Instantaneous Production of Coherent Deuterons

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Abstract

It is possible to produce coherent deuterons by the incidence of coherent photons on solid deuterium. The coherent deuterons, once produced, will fuse to release nuclear energy.

Recently the production of coherent pions in high energy scattering processes has been well studied[1], and there exist some experimental evidences[2] for such coherent pions. Coherent pions, however, are only produced microscopically in small numbers, and hence are of no practical use. In analogy with laser, it was argued[3] that stable charged nuclei such as α-particles and deuterons can be made to become coherent by induced scattering. However charged nuclei, quite unlike photons, are not neutral and interact strongly. Whereas photons can propagate unimpeded in media like glass and air and so can be made coherently in a gradual way by adding one photon at a time, charged nuclei will interact strongly with any media. Therefore, more complication is expected if we want to create coherent charged nuclei in a gradual way like coherent photons created in a laser tube. It would be best if a process is found whereby charged nuclei can be created coherently in an instantaneous fashion so that there is no need for them to travel in any media. Such an instantaneous process is in fact possible.

Let us consider the elementary process of ionizing a deuterium atom D by one energetic photon to become a deuteron d and an electron e:

$$\gamma(\vec{k}) + \mathrm{D}(\vec{p}) \rightarrow \mathrm{e}^-(\vec{q}) + \mathrm{d}^+(\vec{p}') \, , \tag{1}$$

where $\vec{k}$, $\vec{p}$, $\vec{q}$ and $\vec{p}'$ are the momenta of the particles γ, D, e, and d respectively. The effective interaction Hamiltonian for the ionization process (1) is given by

$$H_i = g \int d^3x (A\psi_{\mathrm{D}}\psi_{\mathrm{e}}^*\phi_{\mathrm{d}}^* + \mathrm{h.c.}) \, , \tag{2}$$

where ψ_{D}, ψ_{e}, ϕ_{d} and A are the quantum fields of deuterium, electron, deuteron and photons respectively. All spins are neglected. The effective coupling g can be evaluated from the ionization cross section:

$$\sigma_i = \frac{1}{2\pi\omega} g^2 \mu \Big(\frac{V_\gamma V_{\mathrm{D}}}{V_{\mathrm{e}} V_{\mathrm{d}}}\Big)^{1/4} \Big[2\mu\Big(\omega + \frac{\vec{p}^2}{2m} - \mathcal{E}_{\mathrm{D}}\Big)\Big]^{1/2} \, , \tag{3}$$

where μ, m are the masses of electron and deuteron, ω is the energy of the photon, and $\mathcal{E}_{\mathrm{D}}$ is the ionization energy of the deuterium atom. The normalization volumes $V_{\gamma,\mathrm{D,e,d}}$ for the four different particles γ, D, e and d are given by conditions in the experiment. When the deuterium atom is embedded in a solid block of deuterium, there is a distribution in

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momentum p due to the finite temperature. The transition rate for process (1) to happen has to be averaged over the initial momentum distribution of the deuterium atoms,

$$\bar{w}_1 = \sum_{\vec{p}\vec{q}} 2\pi\delta(\Delta E)|\langle \vec{p}'\vec{q}|H_i|\vec{p}\vec{k}\rangle|^2 N(p) . \tag{4}$$

The distribution is assumed to be Maxwellian:

$$N(p) = \frac{(2\pi)^3}{V_\mathrm{D}}(2\pi m k_B T)^{3/2} e^{-p^2/2mk_BT} , \tag{5}$$

where k_B is the Boltzmann constant, and T is the temperature. Then the averaged rate is

$$\bar{w}_1 = q^2 \mu m k_B T[N(p_0) - N(p_1)]\left[4\pi\omega|\vec{p}' - \vec{k}|\sqrt{V_\gamma V_\mathrm{d}}\right]^{-1} , \tag{6}$$

where p_0 and p_1 are the limits of the rate of momenta that the initial deuterium $D(p)$ can have from conservation of energy-momentum so as to produce the same final momentum of deuteron p',

$$\begin{aligned} p_0 &= \left|\, |\vec{p}' - \vec{k}| - \left[2\mu\left(\omega - \frac{\vec{p}'^2}{2m} - \mathcal{E}_\mathrm{D}\right)\right]^{1/2} \right| , \\ p_1 &= |\vec{p}' - \vec{k}| + \left[2\mu\left(\omega - \frac{\vec{p}'^2}{2m} - \mathcal{E}_\mathrm{D}\right)\right]^{1/2} . \end{aligned} \tag{7}$$

We are interested in producing all deuterons in the same quantum state, out of all the possible final states, which is given by $1/\eta$:

$$\begin{aligned} \frac{1}{\eta} &= \frac{2\pi}{\tau}\sum_{\vec{p},\vec{q}} N(p)\delta_{\vec{p}+\vec{k},\vec{p}'+\vec{q}}\,\delta\left(\omega + \frac{\vec{p}^2}{2m} - \mathcal{E}_\mathrm{D} - \frac{\vec{p}'^2}{2m} - \frac{\vec{q}^2}{2\mu}\right) \\ &= \frac{1}{\tau}\sqrt{\frac{V_\mathrm{e}}{V_\mathrm{D}}}\frac{\mu}{|\vec{p}' - \vec{k}|}\left(\frac{2\pi}{mk_BT}\right)^{1/2}\left(e^{-p_0^2/2mk_BT} - e^{-p_1^2/2mk_BT}\right) , \end{aligned} \tag{8}$$

where τ is the characteristic time of interaction during the scattering process. The probability of process (1) to occur is

$$P_1 = \frac{1}{4} w_1 \tau . \tag{9}$$

Let us now consider the simultaneous ionization of n deuterium atoms by n coherent photons:

$$n\gamma(\vec{k}) + \mathrm{D}(\vec{p}_1) + \cdots + \mathrm{D}(\vec{p}_n) \to n\mathrm{d}^+(\vec{p}') + \mathrm{e}^-(\vec{q}_1) + \cdots + \mathrm{e}^-(\vec{q}_n) \tag{10}$$

with the production of n coherent deuterons with the same momentum $\vec{p}'$. The electrons are produced with a distribution of different momenta $\vec{q}_1, \vec{q}_2, \cdots, \vec{q}_n$. Its transition rate can be calculated through an n-th order perturbation theory[1]:

$$\bar{w}_n = \left(\frac{1}{n!}\right)^2 \sum_{\substack{\vec{p}_1\cdots\vec{p}_n \\ \vec{q}_1\cdots\vec{q}_n}} N(p_1)\cdots N(p_n) 2\pi\delta(\Delta E)\left|\langle n\vec{p}', \vec{q}_1\cdots\vec{q}_n|H_i\left(\frac{1}{\Delta E}H_i\right)^{n-1}|\vec{p}_1\cdots\vec{p}_n, n\vec{k}\rangle\right|^2 , \tag{11}$$

where the bar over $\bar{w}_n$ indicates an average over initial momentum distribution of deuteriums due to finite temperature. Then it can be evaluated to yield

$$\bar{w}_n = (n!)^3 (p_1)^{n-1} \bar{w}_1 Q_n . \tag{12}$$

This expression has obvious physical meaning. For each species of coherent bosons, we have an $n!$ factor due to

$$a_k \cdots a_k |n(k)\rangle = \sqrt{n!}\,|0\rangle , \tag{13}$$

where a_k is an annihilation operator of boson with momentum k. There are two species of n coherent bosons: photons and deuterons in process (10). An additional $n!$ comes from the bosonic character of the commutator of the Hamiltonian. There are n elementary processes, $\gamma D \rightarrow d^+e$ going on, so that the probability is proportional to P_1^n or transition rate is proportional to $P_1^{n-1}\bar{w}_1$. The Q_n comes from the fermionic character of the electron in the final state. The expression for Q_n can be greatly simplified to yield

$$Q_n = \bar{e}^{n(n-1)\eta'/4} , \tag{14}$$

if the following condition is satisfied:

$$n\eta' < 1 \tag{15}$$

with

$$\frac{1}{\eta'} = \frac{\sqrt{V_D V_e}\mu m k_B T}{2\pi\tau|\vec{p}-\vec{k}|} . \tag{16}$$

The physical meaning of Eq. (16) is that the number of states $1/\eta'$ electrons can occupy cannot be exceeded by the number of electrons n due to fermi statistics. The average transition rate $\bar{w}$ can be recasted in the following form:

$$\bar{w}_n = Z^n \bar{w}_1 , \tag{17}$$

where Z is approximately given by

$$Z = \frac{n^3}{e^3} P_1 e^{-n\eta'/4} , \tag{18}$$

where the Stirling formula for $n! = (n/e)^n$ has been used. The exponential damping factor comes from the fact that no two fermions can occupy the same final state. The critical condition for the instantaneous creation of n coherent deuterons is determined by the inequality

$$Z > 1 . \tag{19}$$

When $Z < 1$, the transition rate for n deuteriums to become n coherent deuterons is extremely small because n is a large number n ($> 10^{10}$) and Z^n is very small indeed. On the other hand, if $Z > 1$, the transition rate for n deuteriums to become n coherent deuterons will become very big, and the production can be regarded as instantaneous. The initial condition $Z = 1$ is similar to what occurs in the phase transition in condensed matter.

Finally, we give some numerical estimates. Let us assume a solid deuterium pellet with volume

$$V_D = V_d = V_e = 100\,\mu\text{m} \times 100\,\mu\text{m} \times 100\,\mu\text{m}$$

to be incident by n coherent photons which are focused to the area of the deuterium pellet which is 100μm $\times 100\mu$m. Its pulse length is 100 cm. Therefore, the normalization value of the photon is

$$V \doteq 100\,\mu\text{m} \times 100\,\mu\text{m} \times 100\,\text{cm} .$$

The energy of the photon ranges from

$$E_\gamma = \omega = 14 \text{ to } 22 \text{ eV} . \tag{20}$$

The coupling constant g is calculated using the ionization cross section[5] $\sigma_i = 10^{-17}$ cm^2. From $Z = 1$, we can calculate the minimum number of photon n that is needed for temperature $T = 1°$ to $10°$ K. It is in the range of $n \sim 10^{12}$ as shown in Fig. 1, which means an energy of a μJ per pulse, which is not stringent at all. The value of $n\eta'$ is calculated and shown in Fig. 2 and is smaller than 1, as required. We conclude the parameters required to produce coherent deuterons instantaneously are within the reach of present experimental situations. The immediate use of coherent deuterons is that they will fuse to release an enormous amount of nuclear energy[6].

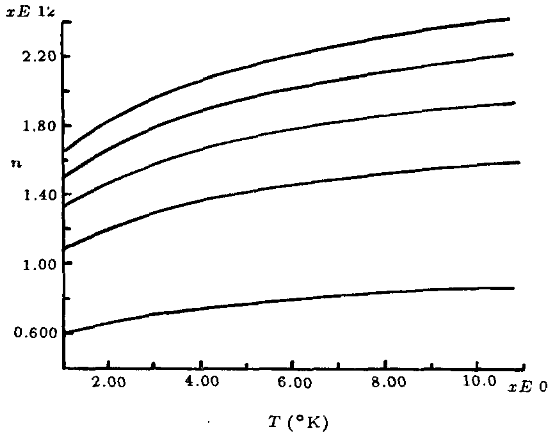

Fig. 1. The minimum number of photons required to satisfy critical transition $Z = 1$ for temperature $T = 2°$ to $10°$ K. The five curves are evaluated for different energies of photon $E_\gamma = 14, 16, 18, 20, 22$ eV from bottom to top.

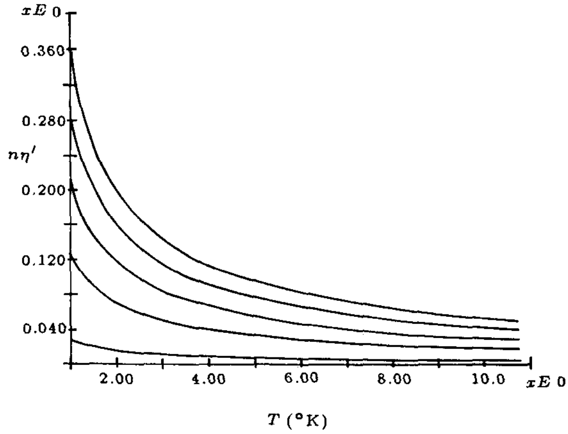

Fig. 2. The values of $n\eta$ at temperature between $2°$ to $10°$ K for different energies of photon $E_\gamma = 14, 16, 18, 20, 22$ eV from bottom to top.

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[4] In the normal S-matrix scattering, both the time τ and normalization volume V are taken to be infinite $\tau \to \infty$, $V \to \infty$. In order for laser to work, it is necessary to count the number of final states available. A finite normalization volume as given by the volume of the laser is needed. In the case of producing coherent bosons in a scattering process as considered here, it is further necessary to use a finite time interval τ as given by the interaction time in order to produce a finite number of final states. We here use $\tau = 2\pi\delta(0) = 1\,\mu$m.

[5] N. Wainfan, W.C. Walker and G.L. Weissler, Phys. Rev. **99**(1955)542. The ionization cross section is considerably bigger than the elastic cross section at these energies. There are, therefore, no other competing channels.

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