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A simple argument that small hydrogen may exist

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ABSTRACT

This paper discusses a possible existence of small hydrogen, which may have been created during the Big Bang before formation of normal hydrogen.

0. Introduction

Rutherford suggested already in 1920 that electron-proton could be bound in tight state [1]. At that point neither the Shroedinger equation (1926) nor Dirac equation (1928) was known to him. He asked his team, including Chadwick, to search for this atom. After Chadwick's discovery of the neutron in 1932 there was a lot of discussions whether it is an elementary particle or a hydrogen-like atom formed from electron and proton [2]. For example, Heisenberg was among those who argued that Chadwick's particle is a small hydrogen atom. At the end the Pauli's argument won, that the neutron spin 1/2 follows Fermi-Dirac statistics and this decided that the neutron is indeed an elementary particle. **This is a well-established fact and it is not discussed in this paper**.

It must have been obvious to both Schroedinger and Dirac, and certainly to Heisenberg, that there is a peculiar solution to their equations. This particular solution, which corresponds to the small hydrogen, was at the end rejected [3] because the wave function is infinite at $r = 0$. Since nobody has observed it, the idea of the small hydrogen has died. However, its idea was revived again ∼70-years later, where authors argued that the proton has a finite size, and that the electron experiences a different non-Coulomb potential at very small radius [4,5]. In fact, such non-Coulomb potentials, for example, Smith-Johnson or Nix potentials [6,7], are used in relativistic Hartree-Fock calculations for very heavy atoms where inner shell electrons are close to nucleus. Using this method, authors retained solutions for the small hydrogen which were previously rejected. However, in a follow up paper [8], it was recognized that considering such potentials does not satisfy Virial theorem, and that one needs to add much stronger potential to hold the relativistic electron stable.

Brodsky pointed out that one should not use the "1930 quantum mechanics" to solve the problem of the small hydrogen; instead, one should use the Salpeter-Bethe QED theory [9]. Spence and Vary attempted to find such electron-proton bound state using QED theory [10], which includes spin-spin, field retardation term and Coulomb potential, assuming the point-like proton. They suggest a possible existence of a bound state.

There are two reasons why the small hydrogen idea was not investigated theoretically further: (a) nobody has found it experimentally, and (b) the correct relativistic QED theory is too complicated at small distances.[1]

Our approach is a potential-based calculation. We propose to solve the problem using a simple equivalent model based on two basic physics principles:

(a) Virial theorem, which is important consideration to judge a stability of bound systems. This requires to think in terms of attractive potentials and electron kinetic energy.
(b) DeBroglie's classical quantum mechanics principle,

stating that the only allowed atomic states are those with integral number of electron wavelengths on a given atomic orbit.

These two assumptions are sufficient to derive energy levels of the normal hydrogen. We will make an **ansatz** that they can be used for the small hydrogen problem also.

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[1] Private communication with Prof. James Vary, one of the authors of Ref. [10].

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One can ask a question what is the small hydrogen? To us, it is not a ground state of the normal hydrogen. Instead, it is a small relativistic electromagnetic vortex of two charges.

1. Simple argument for the small hydrogen

1.1. Balance of two forces

We are not trying to solve the problem by the QED theory; instead we will use potential-based approach based on balancing of two forces, one trying to separate electron-proton pair and the other one trying to bring them closer.

We assume that proton charge is point-like, and we neglect spin-spin effects. We assume that the attractive potential is dominated by a sum of two terms, the Coulomb potential V_{Coulomb} and the Dirac spin-orbit term $V_{(\text{Spin.B})}$ (both potentials are already used for the normal hydrogen,[2] $V_{\text{Coulomb}} = -KZe^2/r$ explains its stability, and $V_{(\text{Spin.B})}$ potential explains the hyperfine spectral structure, which is a tiny effect for electron at large radius).

In this paper, we **assume** that the $V_{(\text{Spin.B})}$ potential can also be used at small radius.

We assume that the spin-orbit potential is:

$$V_{(\text{Spin.B})} \sim -(e\hbar/2mc)\,(\boldsymbol{\sigma} \cdot \boldsymbol{B}) = -\mu_0 \boldsymbol{B}, \tag{1}$$

where $\mu_0 = 5.788 \times 10^{-9}$ eV/Gauss is the Bohr magneton, $\boldsymbol{B}$ is electron "self-induced" magnetic field. To understand the origin of this magnetic field, we shall assume a simple equivalent model, where the electron is considered to be at rest and the proton is moving around at this radius.[3] One can estimate the magnetic field value as follows:

$$\boldsymbol{B} \sim 10^{-7} 2\pi \boldsymbol{I}/r = 10^{-7} Ze\boldsymbol{v}/r^2 \tag{2}$$

where $\boldsymbol{I}$ is the circular loop current, Z is atomic number and $\boldsymbol{v}$ is electron velocity. Magnetic field, calculated using equation (2), at a distance of a few Fermi is extremely high. For example, we obtain $\boldsymbol{B} \sim 5.977 \times 10^{15}$ Gauss at radius of $r \sim 2.8328$ Fermi, making the spin-orbit term $V_{(\text{Spin.B})} \sim 34.5961$ MeV, while the Coulomb contribution is only $V_{\text{Coulomb}} = -KZe^2/r \sim 0.508326$ MeV at the same radius. Fig. 1 shows V_{Coulomb} and $V_{(\text{spin.B})}$ potential shapes as a function of radius close to proton.

Although this paper uses electron radius $\boldsymbol{r}$ in the following formulas, it should be looked at from quantum mechanical point of view, i.e., electron has a distribution of radii with some mean value of $<\boldsymbol{r}>$, determined by its wave function.

1.2. Electron kinetic energy

The kinetic energy T_{kinetic} of an electron located at radius r can be simply estimated as follows:

$$T_{\text{kinetic}} = \sqrt{(hc/\lambda)^2 + (mc)^2} - mc^2 \tag{3}$$

where $\lambda = (2\pi r/n)$ is De Broglie wavelength for electron radius r, n is number of wavelength periods. Fig. 2 shows T_{kinetic} variable as a function of electron radius, and its relationship to absolute values of attractive potentials involved. We can see that the Coulomb potential alone cannot hold electron on a stable orbit for radii below ~10000 Fermi, and that the $V_{(\text{Spin.B})}$ potential is essential to hold the small hydrogen together.

[2] For example, one could use Salpeter-Bethe equation [11].

[3] Such calculation is used in many text books. For example, P.A. Tipler [12] used a similar approach to calculate the spin-orbit fine-structure splitting of spectral lines in the normal hydrogen atom, where he obtains the self-induced magnetic field of $B \sim 4 \times 10^3$ Gauss at $r \sim 2.12$ Å.

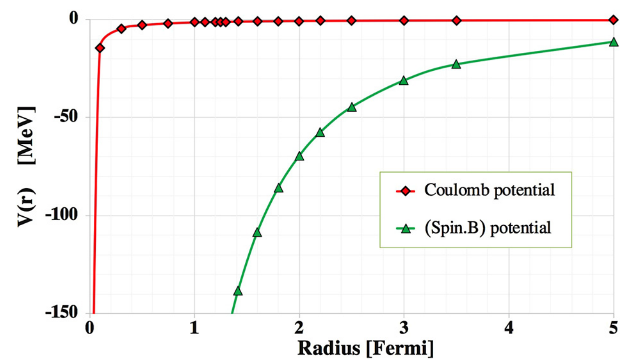

Fig. 1. V_{Coulomb} and $V_{(\text{spin.B})}$ potential shapes as a function of radius close to proton.

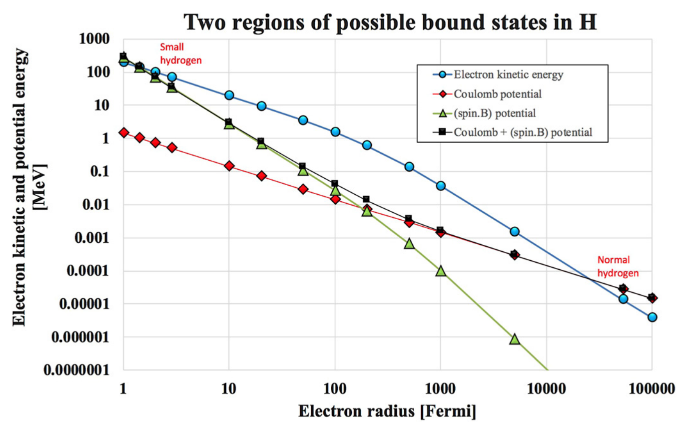

Fig. 2. Comparison of electron kinetic energy T_{kinetic}, and absolute values of V_{Coulomb}, $V_{(\text{Spin.B})}$ and ($V_{\text{Coulomb}} + V_{(\text{Spin.B})}$) potentials.

1.3. Virial theorem

Virial theorem is important consideration to judge a stability of bound systems of two or more particles. Virial relations can be used to draw conclusions on the dynamics of bound states without solving the equations of motion. We will use three independent methods.

1.3.1. Method A

Virial theorem states that for a general potential $V(r) = \alpha r^k$, the expected electron kinetic energy T_{virial} is related to potential energy U as follows [13–15]:

$$T_{\text{virial}} = k\,[\gamma/(\gamma+1)]\,U, \quad \text{where } \gamma = 1/\sqrt{1-(v/c)^2} \tag{4}$$

For the small hydrogen, potential U has two terms, U_1 and U_2. For Coulomb potential ($U_1 = -KZe^2/r$) $k = -1$, and the kinetic virial energy is behaving as $T_{\text{virial}} \to -(1/2)U_1$ as $\gamma \to 1$, and as $T_{\text{virial}} \to U_1$ as $\gamma \to \infty$. For the spin-potential ($U_2 = V_{(\text{spin.B})} \sim 1/r^2$) $k = -2$, and the kinetic virial energy is behaving as $T_{\text{virial}} \to -2U_2$ as $\gamma \to \infty$. Therefore, the virial kinetic energy is:

$$T_{\text{virial}} = k_1\,[\gamma/(\gamma+1)]\,U_1 + k_2\,[\gamma/(\gamma+1)]\,U_2, \tag{5}$$

where $k_1 = -1$, $k_2 = -2$

The condition for stability is:

$$T_{\text{kinetic}} = T_{\text{virial}} \tag{6}$$

Fig. 3 shows that the equation (6), with input from equations (4)&(5), is satisfied for the normal hydrogen atom exactly, in this case using the Bohr model. One can also prove that the

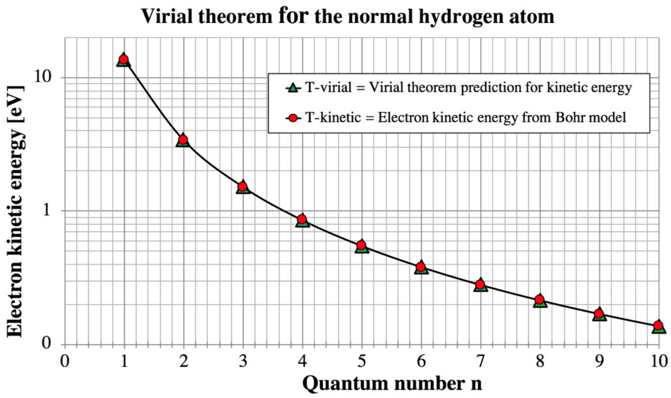

Fig. 3. This plot validates equation (6) for the normal hydrogen, where T_{kinetic} is calculated using equation (3) and T_{virial} is calculated using equation (5).

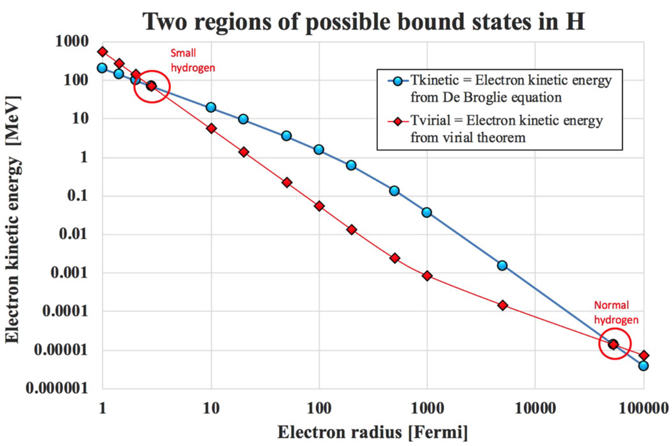

Fig. 4. Comparison of electron kinetic energy T_{kinetic} (equation (3)) and T_{virial} (equation (5)) shows two regions of hydrogen atom stability, one for normal hydrogen and one for the small hydrogen.

Schrodinger's hydrogen satisfies the virial theorem using mean values of Coulomb potential $<\boldsymbol{V}>$ and mean radius $<\boldsymbol{r}>$, obtained from electron wave function.

We will **assume** that equations (4)–(6) are also valid for the small hydrogen atom. Fig. 4 shows that there are two regions where T_{kinetic} is equal to T_{virial}, i.e., where the virial theorem is satisfied.

1.3.2. Method B

Another way to work out the virial theorem is to follow references [13,14,16]. They show that the following equation describes the relativistic virial theorem for a particle moving in potential $U(r)$, when averages over time:

$$<\boldsymbol{p}\partial/\partial\boldsymbol{p}T_{\text{kinetic}}(p) - \boldsymbol{r}\partial/\partial\boldsymbol{r}U(r)> = 0 \qquad (7)$$

where $\boldsymbol{p}$ is electron relativistic momentum, $\boldsymbol{r}$ is electron radius, and $U = V_{\text{Coulomb}} + V_{(\text{Spin.B})}$. One can rewrite eq. (7) using relativistic formulas as follows (we drop averaging over time since we are dealing with a periodic motion):

$$\left((\boldsymbol{p}c)^2/\sqrt{(\boldsymbol{p}c)^2 + (mc^2)^2} - \boldsymbol{r}\partial/\partial\boldsymbol{r}(V_{\text{Coulomb}} + V_{(\text{spin.B})})\right) = 0 \qquad (8)$$

One can solve this equation numerically and result is shown on Fig. 5, where we plot an absolute value of the left side of eq. (8) as a function of electron radius. The result agrees exactly with eq. (6), i.e., the stability occurs at $r \sim 2.83$ Fermi for $n = 1$.

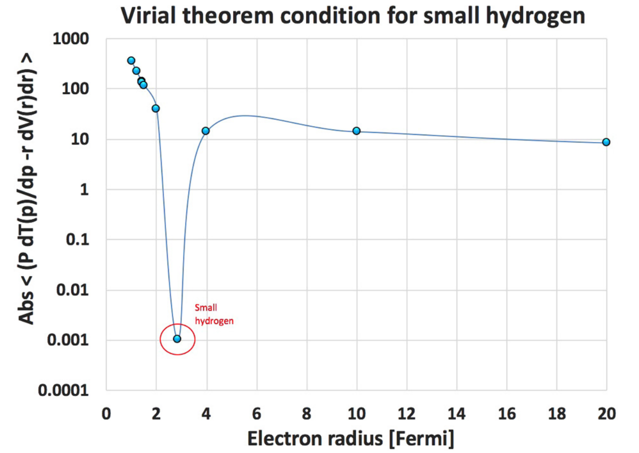

Fig. 5. Numerical solution of equation (8) for $n = 1$. The virial theorem condition for stability occurs at $r \sim 2.83$ Fermi. The curve does not reach zero because of a finite binning.

Table 1
Small hydrogen solutions ($r = r_{\text{min}}$).

n	Radius r [Fermi]	$V_{(\text{Spin.B})}$ [MeV]	T_{kinetic} [MeV]	$E_{\text{Binding energy}}$ [MeV]	Mass* [MeV/c^2]
1	2.8328	−34.5961	69.1923	−0.25416	938.52892
2	1.4113	−139.3929	279.3056	−0.51017	938.2729

* Mass of small hydrogen $= m_{\text{proton}} + m_{\text{electron}} - E_{\text{binding energy}}$. (Mass of neutron = 939.565413 MeV/c^2.)

1.3.3. Method C

One can reach the same stability conclusion by searching for a minimum in total electron energy $E = T_{\text{kinetic}} + U$, i.e., searching for a radius where the following equation is valid:

$$d\boldsymbol{E}/d\boldsymbol{r} = d\left(T_{\text{kinetic}} - Abs(V_{\text{Coulomb}} + V_{(\text{Spin.B})})\right)/d\boldsymbol{r} = 0 \qquad (9)$$

We will use a numerical method to solve the equation (9). The solutions are shown as minima of curves on Fig. 6, where dE/dr is equal to zero. We present two curves for $n = 1\&2$, where n is integral number of waves on a given orbit in the De Broglie equation ($n\lambda = 2\pi r$). Looking at these curves, one can assign a radius r_{min} corresponding to minimum of each curve. It turns out that this radius corresponds to solutions of equations (6) and (8) **exactly**.

Looking at Fig. 6, one can also assign a mean value $<\boldsymbol{r}>$ to each curve, which tends to be larger than r_{min}, and a width $\boldsymbol{\Delta x}$, which describes how broad each minimum is (we estimate $<\boldsymbol{r}>\sim$ 5–6 Fermi and $\Delta x \sim$ 10–20 Fermi for $n = 1$). Table 1 shows several other variables calculated for the condition of stability of the small hydrogen using equations (6), (8) and (9).

As n gets larger, electron radius gets smaller. In fact, for $n \geq 3$, the radius is smaller than radius of proton (∼1 Fermi), and such solutions were disregarded. If a transition "$n = 2 \rightarrow 1$" exists, emitted photon would have energy of ∼256 keV. Transitions between normal hydrogen and two small hydrogen levels, if they can be triggered somehow, could produce photons of ∼254.16 and 510.17 keV.

One should point out that the virial theorem is satisfied only for one orientation of spin, i.e., when spin flips and $V_{(\text{Spin.B})}$ changes a sign, such configuration is unstable.

Our calculation also shows that other fully ionized "small-Z atoms" can form small-radius atoms, if incident electron beam has appropriate energy. This would create atoms, where one electron is trapped on a small radius, effectively shielding one proton charge of nucleus, thus making the atom behaving chemically and spectroscopically almost as ($Z - 1$)-atom. Fig. 7 shows stable states for

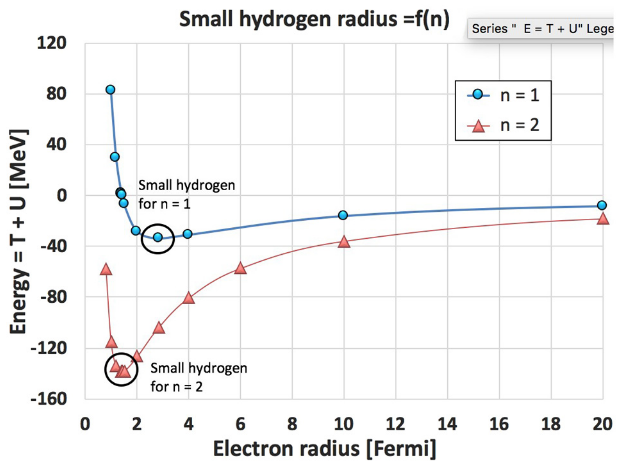

Fig. 6. Total electron energy $E = T + U$ as a function of electron radius for $n = 1\&2$ in the De Broglie equation ($n\lambda = 2\pi r$). Minimum of each curve ($r = r_{\min}$) is a solution of equation (9); this minimum agrees exactly with solutions of equations (6) and (8).

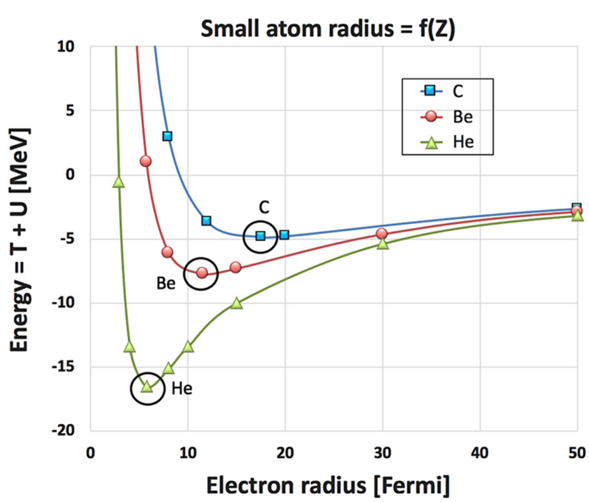

Fig. 7. Total electron energy $E = T + U$ as a function of electron radius for a single electron on a "small orbit" in fully ionized atoms of He, Be, and C.

He, Be and C atoms. One can see that curve minimum widths are considerably wider.

2. Heisenberg uncertainty principle

It is interesting to note that Heisenberg published his uncertainty principle paper in 1927, and yet he still supported the idea that Chadwick's neutron is a small hydrogen in 1934 [2], i.e., he did not reject it from point of view of the uncertainty principle ($\Delta x \Delta p \geq \hbar/2$). According to this principle, tighter electron is confined in space, broader energy distribution it will have. It must have been clear to Heisenberg that the only way to confine electron in a tight space it has to be held with adequate attractive force.

This paper considers electron to be outside of nucleus. Still the electron is relativistic. We argue that the only way to achieve stability is to use the $V_{(\mathrm{Spin.B})}$ potential at small radius.

Although this paper uses electron radius in formulas (4)–(9), it should be looked at from a point of view of quantum mechanics, i.e., electron has a certain distribution of radii with some mean value of $< r >$. Fig. 6 shows that radius distribution is very broad; although radius corresponding to curve minimum is small ($r_{\min} \sim 2.8328$ Fermi for $n = 1$), one could assign a mean radius of $< r > \sim 5 - 6$ Fermi and a width of the distribution $\Delta x \sim$ 10–20 Fermi, which would correspond to $\Delta p \geq$ 5–10 MeV/c, according to the uncertainty principle. This would mean that transitions to the small hydrogen may have large width.

3. Do small atoms exist?

To form the normal hydrogen, a free electron needs to be almost at rest relatively to proton. Once such atom is formed, electron will spontaneously cascade down to ground level through allowed transitions, releasing photons with total released photon energy of up to 13.6 eV. The important point is that electrons on all these levels are non-relativistic and energy differences are very small. The experimental evidence shows, that one never forms the small hydrogen spontaneously, because we do not observe high energy photons (254.16 or 510.17 keV). This indicates that such transitions either do not exist because the small hydrogen does not exist, or they are forbidden, or they are very unlikely. We argue that a spontaneous transition from normal level to small level is unlikely because of a large electron energy difference in both states.

The small hydrogen may be formed differently; for example, using a relativistic electron with a correct wavelength latching on a proton. Such condition may have occurred during the Big Bang, or during other very energetic and luminous events in the Universe. One could try to use a high intensity electron beam of precisely tuned energy, and look for a sign of e-p bonded state formation. If the small hydrogen is formed, it would appear as a neutral object from some distance. Such object might be able to enter the boron nucleus in boron-based detectors, destabilize the nucleus, which may produce alpha particle, which then would be detected. However, this process might turn out to be very unlikely because the small hydrogen does have a fairly large size compared to nucleus size, and it has an electric dipole moment, which may prevent entry into the nucleus.

Another avenue is to search for $(Z - 1)$-atoms. They would not form spontaneously. It would require a dedicated experiment to find them. A high intensity electron beam could be used to irradiate neutral atoms, for example, helium, and completely strip it of electron-shell first. If one high energy electron is latched on a small radius, and the second one on a normal level, new neutral atom would behave chemically and spectroscopically almost as a heavy hydrogen. Similarly, beryllium would behave as lithium, and carbon as boron, etc. However, there is a limit for stability of such atoms as $V_{(\mathrm{Spin.B})}$ gets weaker at larger radius. For example, we have determined that an electron on fully ionized xenon does not satisfy equations (6)–(9) any more. Since it is difficult to latch an electron on small orbit, a sample of such $(Z - 1)$-atoms would represent a "small contamination," and it would not surprise that nobody has noticed them.

4. Astrophysics implications

The small hydrogen, if it exists, will interact gravitationally mainly. Otherwise it would have a little interaction, meaning almost negligible dE/dx deposit due to its tiny electric dipole moment and feeble nuclear interactions as it may have some difficulties to enter nuclei. It would also be difficult to find it in typical astrophysical spectroscopic observations. One could search for the 256 keV line, but it may be broad.

If the small hydrogen has been produced in the Big Bang, it may have created the web-like structure of the Universe. The small hydrogen could also be formed in extreme cosmic events within a galaxy. As a galaxy gets older, it would accumulate more and more of this type of matter, which would influence progressively the orbital velocity of visible matter within the galaxy.

Since it would be impossible to see the small hydrogen directly using spectroscopic techniques, it would appear to us as the dark matter.

5. Conclusion

This paper has suggested, using simple semi-classical potential-based arguments, that there exist a sufficient force to hold proton and electron together to form the small hydrogen, which we consider as a small vortex of two oscillating relativistic charges. This may motivate experimental searches and efforts to pursue more advanced QED calculation. If such small hydrogen exists it would have a significant impact on astrophysics.

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