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On a Possible Wave Mechanism for an Increase in the Yields of Low-Energy Nuclear Reactions in Crystal Structures

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Abstract—The possibility of using one of the properties of wave propagation in periodic structures is considered to explain the known effect of an increase in the yields of nuclear DD reactions in crystalline films. The effect is observed under the bombardment of deuterated metal foil targets with an oxide film by low-energy deuterons (less than 100 keV) in the majority of metals: the yield increases many times as compared with the expected yield when extrapolating the data obtained for high-energy deuterons. Proceeding from the analogy between equations describing the propagation of a light wave in a photonic crystal and that of a particle in a crystal, the conclusion that the properties of the wave functions of particles in a crystal and of the light field in a photonic crystal are analogous is drawn. Forbidden bands with transparency windows can exist in the spectrum of the particles; their properties are also analogous to those of the windows in a photonic crystal. The amplitude of the wave function of a massive particle incident on a crystal can increase in them, which must be taken into account when considering the increase in the yields of nuclear reactions with the participation of moving particles.

Keywords: nuclear reactions, crystals, increase in the yield, deuterons

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INTRODUCTION

It was mentioned in publications (for example, [1, 2]) that the measurement of cross sections for nuclear-fusion reactions at low energies (at most 100 keV) is of significant interest, and because the direct measurement of cross sections is difficult in this range, they are calculated via extrapolation from the high-energy range, where they are measured using accelerators. In experiments using accelerators with low energies (less than 10 keV) and solid targets with implanted deuterium, a significant increase in the yield of the DD reaction is observed as compared with data obtained by extrapolation from the high-energy range. The authors of [3, 4] studied more than 70 elements of the periodic system, and this effect was discovered in the majority of studied metals, except for six. A larger increasement of the yield of this reaction was observed at higher current densities and lower ion energies (0.8–2.5 keV) [5], and the coefficient of increase in the yield of this reaction was 10^9 [6], when ions energy was 1.0 keV.

The authors of [1] used a Pd foil target with an oxide film (PdO) on the surface. The samples were saturated with deuterium before the experiment. The authors of [2] used Ti foil samples with an oxide layer, which were also saturated with deuterium. The ion energy ranges from 10 to 25 keV. In both materials, a growth of the coefficient of increase in the DD-reaction yield was observed with decreasing ion energy. At 10 keV, the yield increased twofold in the Ti target and fourfold in the Pd one. Thus, the effect was due to collisions between injected deuterons and deuterons absorbed by the crystal lattice. This effect depends on the target orientation, namely, yields can differ by several times in the case of different orientations. These data, together with the data in [3–6], led to the necessity to consider the important aspect of wave propagation in periodic structures, which can play a large role in an increase in the yields of similar reactions for deuterons.

The aim of this paper is to propose the mechanism for an increase in the wave-function amplitude of incident deuterons inside the crystal, which is well known in the optics of the visible, IR- (infrared) and radio frequency ranges, i.e., the effect related to the motion of an incident deuteron in a periodic crystal field. All questions concerning nuclear reactions are beyond the scope of this paper.

COMPARISON OF THE PROPERTIES OF THE PROPAGATION OF A MASSIVE PARTICLE WITH THAT OF LIGHT IN A PERIODIC MEDIUM

In this paper, we deal with a possible increase in the concentration of incident deuterons inside a crystal (compared with a vacuum) at definite energies (and, accordingly, de Broglie wavelengths). The problems of changes in the concentration of particle targets absorbed by a crystal lattice are not considered here. We proceed from the property of an increase in the amplitude of the incident light wave at certain energies in a photonic crystal (this was first considered in [7] and was then developed, for example, in [8]). The transmission spectrum of the light emission of such a crystal has a series of forbidden bands with centers corresponding to the resonance Bragg reflection, inside which emission barely propagates. Resonance occurs if the wavelength in the medium (or its harmonic, i.e., the wavelength decreases by a whole number of times) coincides with the corresponding doubled lattice period. A series of total transmission maxima called transparency windows are observed for a photonic crystal with a finite thickness, beyond the forbidden bands, near their boundaries, in the absence of losses caused by absorption and light scattering. The calculation shows that the increase in the light intensity inside the crystal can reach many orders at the wavelength corresponding to total light transmission. This is confirmed by a series of experiments, in particular, in cases of the generation of the second harmonic [9] and of the spontaneous [10] and stimulated Raman scattering processes [11].

The light field in a one-dimensional photonic crystal is described by the expression:

$$-\frac{1}{\varepsilon(x)}\frac{\partial^2}{\partial x^2}u(x) = \frac{\omega^2}{c^2}u(x), \tag{1}$$

where ω is the frequency, $\varepsilon(x)$ is the permittivity, and $u(x)$ is the complex amplitude of the electric vector of the light field. The wave function $\psi(x)$ of the massive particle (the incident particle) is described by the equation:

$$\left[-\frac{\hbar^2}{2m}\frac{\partial^2}{\partial x^2} + U(x)\right]\psi(x) = E\psi(x), \tag{2}$$

where m is the particle mass, $U(x)$ is the periodic crystal field, and E is the particle energy.

As shown in [7, 8, 12, 13], the intensity of the incident light wave increases strongly in the transparency windows in a photonic crystal, the maximum of the crystal-thickness distribution of the intensity increases as N^2, where N is the number of layers. We consider how close the analogy is between the eigenvalue problems (1) and (2) applied to incident waves (incident particles). As in the case of a photonic crystal (1), nonzero reflection from one layer must occur in the case of a massive particle (2). As in the case of (1), in the case of (2), if the Bragg conditions are satisfied, forbidden bands can exist in the crystal. Both operators on the left-hand-sides of the equations are periodic. In the case of a finite crystal, the joining of its solutions at its boundaries occurs analogously to that in a photonic crystal. Consequently, it can be concluded that it is possible that, for incident massive particles (2), transparency windows also appear near the forbidden band with increasing wave-function amplitude in them, and the character of the increase in the wave-function amplitude is similar to that in the amplitude of the incident light wave in a photonic crystal. The increase in the squared modulus of the wave function of incident particles in the transparency windows means an increase in the probability of the presence of an incident particle at a given point, which can lead to a corresponding increase in the yields of nuclear-fusion reactions [1–6]. We emphasize that the case in point is an increase (compared with the absence of the crystal lattice) in the concentration of incident particles at the same parameters of the incident beam (the same concentration and energy in it) beyond the crystal. This effect depends cardinally on the energy of incident particles and is maximum if it turns out to be in the transparency window [13].

In [13], the dependence (known for the transparency windows [8]) of the coefficient of an increase in the intensity inside the photonic crystal on its parameters was shown in the general form without concretization of the permittivity profile of the photonic crystal and without the determination of eigenfunctions for the field by means of perturbation theory; the dependence of the position and widths of these windows on the same parameters was also shown in this paper. The wave intensity was proportional to the squared number of periods.

Simple estimation showed that, at a period of 0.366 nm, for a relative amplitude of the variable part of the potential of 0.01 and a crystal thickness of 36.6 μm, the increase in the wave function amplitude was 10^3 (compared with the case of absence of the crystal lattice), which corresponded to an increase in the intensity by a factor of 10^6. This increase is of estimation character and is valid only for a very narrow spectral region. Nevertheless, for particles with the corresponding energy in the narrow energy range (falling within the transparency window), it can be asserted that the reaction yields per time unit must be one hundred times larger in the case of a crystal with a thickness that is ten times larger. Though it is beyond the scope of this paper, we note that the considered deuterons (with an energy of 10 keV) are rather energetic for tunneling under the Coulomb barrier. It is possible to estimate the probability of such tunneling, i.e., the ratio of the squared modulus of the wave function before the barrier to that after it (in the region of the nucleus), following the method presented in [14] (p. 218, Problem 2). There the tunneling of a particle with a moment of $l = 0$ was considered (for the nonzero moment l, the wave-function amplitude of the particle decreases toward the center as r^l, and, accordingly, the probability of its position at the reference point coinciding with the nucleus is close to zero). When considering the motion of the particle with $l = 0$, the problem becomes one-dimensional, and the following formula can be used ([14], formula (50,5)):

$$w \sim \exp\left(-\frac{2}{\hbar}\int_{r_0}^{\alpha/E}\sqrt{2m\left(\frac{\alpha}{r}-E\right)}dr\right). \quad (3)$$

Here, r_0 is the nucleus radius; α is the Coulomb interaction constant, $\alpha = Z_1Z_2e^2$; e is the elementary charge; and Z_1e and Z_2e are the charges of the interacting particles. When integrating with the limiting transition $r_0 \to 0$ taken into account, we obtain the following expression for the tunneling probability w:

$$w \sim \exp\left(-\frac{\pi\alpha}{\hbar}\sqrt{\frac{2m}{E}}\right) = \exp\left(-\frac{2\pi\alpha}{\hbar v}\right). \quad (4)$$

These formulas are applicable in the case where the exponent is large, i.e., $\alpha/(\hbar v) >> 1$ (v is the particle velocity). In accordance with formula (4), it is possible to estimate the probability of deuteron tunneling under the Coulomb barrier with the energy E.

The probability is 0.76×10^{-6} for an incident deuteron with an energy of 10 keV and 1.1×10^{-4} for it with an energy of 24 keV. The probability is small, but such "smallness" can be compensated by an increase in the wave-function amplitude because of the presence of the crystal. In the case of optics, the increase in the light-wave amplitude is proportional to the growth in the number of layers; and that in the intensity (an analogue of the squared modulus of the wave function), to the squared number of layers. The above-mentioned estimates give an increase in the squared modulus of the wave function before the barrier by a factor of 10^6. Consequently, comparing the probability of the above-barrier propagation of a deuteron in the absence of a crystal (assuming that this probability of order of unity) and the probability of tunneling of a deuteron with 10 keV energy in a crystal, we see that they are of the same order. To reach such a result in the case of a 24 keV deuteron, the number of lattice periods must be ten times smaller. The lowest energy at which resonance can occur exists if the particle wavelength coincides with the crystal lattice period. For such a period, the de Broglie wavelength is 0.732 nm, which corresponds to a deuteron energy of 7.65×10^{-4} eV and its velocity of 271 m/s. It should be mentioned that this is the main resonance, i.e., a forbidden band with the minimum deuteron energy (or the maximum wavelength), at which the effect can occur. Particles with energies on the order of 10–20 keV are in the range of forbidden bands with rather large energies. However, the effects of an increase can also occur for particles with such energies included in the transparency window. This reasoning is based on the principles in [13]. For short wavelengths, when considering the motion of particles and one potential period, the situation is analogous to the case of the operation of a Fabry–Perot interferometer in optics where the distance between mirrors is much larger than the light wavelength. And as in the case of optics where it is possible to successively install a series of mirrors, which form a composite Fabry–Perot interferometer (which is well known), in the case under consideration, a series of successive periods form a similar structure with the same distances between the boundaries. The transparency windows are located near such bands. However, the conditions of their operation require special consideration.

We note that the above presentation is related to the established stationary state. In the case where a single particle is incident on the crystal, it is necessary to take the dynamic process into account, which can change the process rather quantitatively. This problem requires special consideration. It is also worthwhile to note that the wave-function amplitude can increase in the region of defects in a crystal in much the same way as in a photonic crystal. As an example, we can mention the case of so-called "Tamm states" where the periodicity is violated. Then in a photonic crystal, near a defect even inside the forbidden band, the light-wave intensity can increase stronger than in the transparency window. In [1, 2], an oxide was deposited on the metal foil, i.e., inhomogeneous systems were used. The observed effect could occur in the case of a sharp change in the periodicity. Therefore, the structure of the target layers (single- or polycrystalline, the ratio of the small-crystal dimensions to the layer thickness and the character of their contacts in the latter case) is important to interpret the results by means of this mechanism.

In a three-dimensional crystal, consideration within the framework of the one-dimensional model is possible for a negligibly small coupling of the mode under consideration with others in the case where the particle exchange between these modes is insignificant. In the opposite case, special consideration is required. We note that it is probable that existence of the total forbidden band is not required in the three-dimensional case, i.e., the presence of a band in all directions and the complete forbidding of propagation along all directions at some energy. The presence of the band (or the energy gap) in some direction is sufficient for the effect.

All the above presentation can be related not only to nuclear reactions between deuterons moving inside a solid body and adsorbed by a crystal lattice, but also to those of a deuteron moving inside a solid body with lattice components; the yields of them can also increase. It should be noted that efforts were made previously to explain the increase in the yields of such reactions by means of the Bose condensate of deuterons [15, 16].

CONCLUSION

Thus, in this paper, proceeding from the analogy between the equation describing light propagation without losses in a medium with periodic permittivity and the Schrödinger equation with a periodic potential describing the motion of a massive charged particle inside a crystal lattice, we formulate the assumption that, as in the case of a light field in the transparency windows [7, 8, 12, 13], in the case of a massive particle, the wave-function amplitude can also increase in "transparency windows" that appear located near forbidden bands, which can lead to an increase in the yields of nuclear reactions with the participation of moving particles.

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