Resonance-like nuclear processes in solids: 3rd and 4th order processes

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It is recognized that in the family of heavy charged particle and electron assisted double nuclear processes resonance-like behavior can appear. The transition rates of the heavy particle assisted 3rd-order and electron assisted 4th-order resonance like double nuclear processes are determined. The power of low energy nuclear reactions in $Ni-H$ systems formed in Ni placed in H_2 gas environment is treated. Nuclear power produced by quasi-resonant electron assisted double nuclear processes in these $Ni-H$ systems is calculated. The power obtained tallies with experiments and its magnitude is considerable for practical applications.

I. INTRODUCTION

Since the "cold fusion" publication by Fleischmann and Pons in 1989 [1] a new field of experimental physics has emerged. Although even the possibility of the phenomenon of nuclear fusion at low energies is doubted by many representatives of mainstream physics, the quest for low-energy nuclear reactions (LENR) flourished and hundreds of publications (mostly experimental) have been devoted to various aspects of the problem. (For the summary of experimental observations, the theoretical efforts, and background events see e.g. [2], [3].) The main reasons for revulsion against the topic have been: (a) according to standard knowledge of nuclear physics due to the Coulomb repulsion no nuclear reaction should take place at energies corresponding to room temperature, (b) the observed extra heat attributed to nuclear reactions is not accompanied by the nuclear end products expected from hot fusion experiences, (c) nuclear transmutations were also observed, that considering the Coulomb interaction is an even more inexplicable fact at these energies.

The situation is further complicated by the fact that the electrolysis, gas discharge and/or high pressure gas environments that are stipulated to induce LENR have their effect through rather complex microscopic processes that in most of the cases are difficult to reproduce. As a result, for explaining the riddle of cold fusion it is indispensable to understand theoretically the underlying nuclear reactions.

In our opinion we made progress [4] in the theoretical clarification of the nuclear physics behind LENR. The idea is based on the fact that if a heavy, charged particle (proton, deuteron) of low energy enters a solid (metal), then through the Coulomb interaction it changes the state of charged particles, primarily quasi free valence electrons in the metal while its own state is also changed. Our results of standard perturbation theory calculations indicate that by means of the Coulomb interaction the ingoing charged particle in this change of state can obtain so high value of virtual momentum (energy) that is enough to induce various nuclear reactions including fusions and/or transmutations. With the results of our theoretical work, we found reasonable answers to the above questions (a, b and c).

In [4] however, it was not taken into account that the emergence of new charged particles may alter the state of the solid too in a way that can allow further nuclear processes, such as the family of double nuclear processes which has special interest from the point of view of low energy nuclear processes. In this paper this effect is considered. First a 3rd-order double nuclear process is investigated the graphs of which can be seen in FIG. 1. It is shown that a resonance can appear in this process. Next a 4th-order double nuclear process is discussed the graphs of which can be seen in FIG. 2 which shows resonance-like characters too. Finally the nuclear power produced in $Ni-H$ systems is calculated.

This paper is mainly based on [4]. The notation and the outline of the calculation is the following. The Coulomb coupling strength $e^2 = \alpha_f \hbar c$ and the strong coupling strength $f^2 = 0.08\hbar c$ [5], e is the elementary charge, α_f is the fine structure constant, $\hbar$ is the reduced Planck constant and c is the velocity of light.

When calculating the matrix elements of the strong interaction potential, the long wavelength approximation $|\varphi(\mathbf{0})| = f_{jk}(k)/\sqrt{V}$ of the Coulomb solution $\varphi(\mathbf{r})$ is used, that is valid in the range of a nucleon, where $f_{jk}(k)$ is the appropriate Coulomb factor corresponding to the particles, which take part in strong interaction and V is the volume of normalization. We introduced the following notation

$$F_{jk}(k) = f_{jk}^2(k) = \frac{2\pi\eta_{jk}(k)}{\exp\left[2\pi\eta_{jk}(k)\right] - 1}. \quad (1)$$

Here the Sommerfeld parameter $\eta_{jk}(k)$ for particles j

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and k of electric charge numbers z_j and z_k is determined as

$$\eta_{jk}(k) = z_j z_k \alpha_f \frac{\mu_{jk} c}{\hbar k}, \qquad (2)$$

where $k = |\mathbf{k}_j - \mathbf{k}_k|$ is the magnitude of the relative wave vector $\mathbf{k} = \mathbf{k}_j - \mathbf{k}_k$ of the interacting particles of wave vectors $\mathbf{k}_j$ and $\mathbf{k}_k$ and $\mu_{jk} = m_j m_k / (m_j + m_k)$ is the reduced mass of particles of rest masses m_j and m_k.

For quasi-free particles (electron and ingoing proton) taking part in Coulomb interaction we use plane waves. Thus their Coulomb matrix element is calculated in the Born approximation which is corrected with the so called Sommerfeld factor

$$g_S(k_{in}, k_{out}) = f_{jk}(k_{in}) / f_{jk}(k_{out}) \qquad (3)$$

[6], where k_{in} and k_{out} are the magnitudes of the relative wave numbers before and after Coulomb scattering. For other details and notation see [4].

II.   RESONANCE-LIKE HEAVY CHARGED PARTICLE AND ELECTRON ASSISTED DOUBLE NUCLEAR PROCESSES

Preliminarily one must emphasize that in both (3rd and 4th order) double nuclear processes discussed resonances may occur. The reason for the possibility of resonance is that the continuum of the kinetic energy of an intermediate state is shifted down by the energy of the (first) nuclear transition and therefore one of the denominators in the perturbation calculation can be equal to zero. The occurrence of resonances increases the rate significantly.

A.   Resonance-like heavy charged particle assisted double nuclear processes

First the resonance-like heavy charged particle assisted double nuclear processes (see FIG. 1) are discussed. The particles in FIG. 1 are all heavy, and positively charged. The ingoing particle is particle 2, which belongs to system B (the ensemble of incoming particles forms system B). It is supposed that it has moderately low energy (of about keV order of magnitude), that is raised in the second order processes discussed in [4] or after it in the decelerating process. Particle 2 scatters by Coulomb scattering on particle 1 localized in the solid (system A). Particles 3 and 4 are the nuclear targets of system A. The particles in the intermediate state 1' and 2' may pick up large enough wave vector to overcome the Coulomb repulsion due to particles 4 and 3. Particles 5 and 6 are products of the process. The nuclear process $1' + 4 \rightarrow 5$ takes place as the consequence of the modification of system A by system B. Here both nuclear processes are thought to be nuclear captures. It can be shown (see Appendix I.) that the process may have resonance like character if the
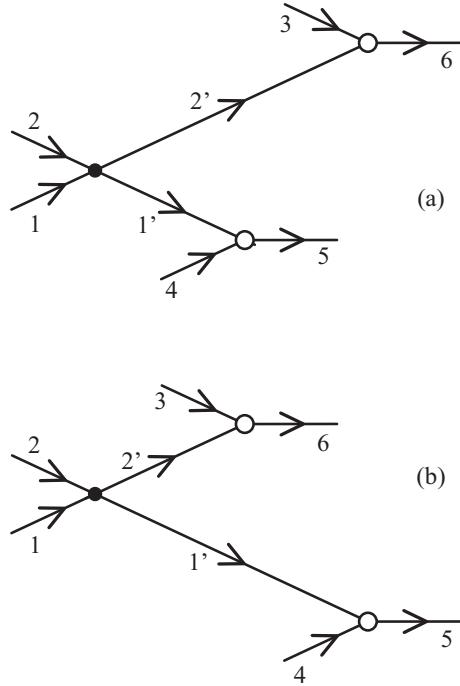

FIG. 1: The graphs of quasi-resonant heavy charged particle Coulomb-assisted nuclear reactions. The filled dot denotes Coulomb-interaction and the open circle denotes nuclear (strong) interaction. Free particle 2 (e.g. p or d) belongs to system B. Localized particles 1, 3 and 4 belong to system A. All the particles are heavy and positively charged. From the point of view of the nuclear process particles 1' and 2' are ingoing, particles 3 and 4 are targets and particles 5 and 6 are reaction products. In the case discussed in the text in the process of FIG. 1(a) there is a resonance, therefore its contribution to the rate is dominant and the contribution of the process of FIG. 1(b) is neglected.

masses of particles 5 and 6 differ significantly, therefore the contribution of the leading graph (e.g. FIG. 1(a) in the case $m_5 \gg m_6$ discussed) to the rate is enough to calculate.

We take as initial state of a localized particle k

$$\psi_{ki}(\mathbf{x}) = \left(\beta_k^2 / \pi\right)^{3/4} \exp\left(-\beta_k^2 \mathbf{x}^2 / 2\right), \qquad (4)$$

which is the ground state of a 3-dimensional harmonic oscillator of energy $E_0 = \frac{3}{2} \hbar \omega_k$ and of angular frequency ω_k [7]. Now we take particle 1 ($k = 1$) localized and the initial state $\psi_{1i}(\mathbf{x})$ is of the form (4) with the parameter $\beta_1 = \sqrt{m_1 \omega_1 / \hbar}$, where m_1 is the rest mass of the localized particle 1. The Coulomb matrix elements which contain initial state of form $\psi_{ki}(\mathbf{x})$ also preserve momentum [4].

The total rate of the reaction

$$\begin{aligned} W_{tot}^{(3)} &= K^{(3)} K_1^{(3)}(\Delta, \Delta_5) h_{corr,3}^2 h_{corr,4}^2 N_{134} \qquad (5) \\ &\times \sum_{k_{2i}} H(k_{2i}, k^r) N_2(k_{2i}), \end{aligned}$$

where N_{134} is the number of triples of particles 1, 3 and 4, that may be effective for one particle 2, and $N_2(k_{2i})$ is the actual number of quasi-free particles 2 of wave number k_{2i}. Furthermore

$$K^{(3)} = 9216\alpha_f^2\pi^{7/2}\left(1 - \frac{2}{e}\right)^4 \left(\frac{f^2}{\hbar c}\right)^4 R^2 c\beta_1^3, \quad (6)$$

where $R = 1.2 \times 10^{-13}\,[cm]$ is the proton-radius,

$$K_1^{(3)}(\Delta, \Delta_5) = \frac{\mu_{12}^2 c^4\sqrt{\mu_{56}c^2\Delta}}{\sqrt{2}\left(\Delta_5 - \frac{\mu_{56}}{m_5}\Delta\right)^3}\frac{\mu_{56}}{m_2}, \quad (7)$$

where $\Delta_5 = \Delta_{01} + \Delta_{04} - \Delta_{05}$ is the energy of reaction $1' + 4 \rightarrow 5$, and $\Delta_6 = \Delta_{02} + \Delta_{03} - \Delta_{06}$ is the energy of reaction $2' + 3 \rightarrow 6$. The Δ_{0j}-s are the energy defects of the corresponding nuclei and $\Delta = \Delta_5 + \Delta_6$ is the total reaction energy.

$$H(k_{2i}, k^r) = g_S^2(k_{2i}, 2k^r)F_{23}(k^r)F_{14}(k^r), \quad (8)$$

where

$$k^r = \frac{\sqrt{2m_2\left(\Delta_5 - \frac{\mu_{56}}{m_5}\Delta\right)}}{\hbar}. \quad (9)$$

$h_{corr,3}$, $h_{corr,4}$ are defined by Eq.(45) of [4] and are determined as

$$h_{corr,k} = A_k - z_k \quad (10)$$

in the case of proton captures, and

$$h_{corr,k} = A_k \quad (11)$$

in the case of deuteron captures. (For the details of the calculation see Appendix. I.)

In a numerical example let particles 1, 2, 3 be deuterons, particle 4 be $^{106}_{46}Pd$, particle 5 be $^{108}_{47}Ag$ and particle 6 be ^4_2He. If particle 1 is a deuteron then $\beta_1 = 4.81 \times 10^8\,[cm^{-1}]$ (see Sec. VIII. A of [4]). With these choices

$$\Delta_6 = 23.847\,[MeV], \Delta_5 = 10.833\,[MeV], \quad (12)$$

$$\Delta = 34.681\,[MeV], \quad (13)$$

$$\mu_{12}c^2 = m_0c^2 = 931.494\,[MeV], \ \mu_{56}c^2 \simeq 4m_0c^2, \quad (14)$$

$$\mu_{56}/m_5 = 1/28, \ \mu_{56}/m_2 \simeq 2. \quad (15)$$

With these numbers one can obtain

$$\Delta_5 - \frac{\mu_{56}}{m_5}\Delta = 9.60\,[MeV], \quad (16)$$

$$K_1^{(3)}(\Delta, \Delta_5) = 5.00 \times 10^5 \quad (17)$$

and

$$\hbar ck^r = 189\,[MeV], \quad (18)$$

the latter producing

$$F_{23}(k^r) = 0.89, F_{14}(k^r) = 2.79 \times 10^{-8}. \quad (19)$$

Finally one gets

$$W_{tot}^{(3)} = 1.45 \times 10^8 \times N_{134}\sum_{k_{2i}} g_S^2(k_{2i}, 2k^r)N_2(k_{2i})\,[s^{-1}]. \quad (20)$$

The $g_S^2(k_{2i}, 2k^r)$ dependence is similar to the $G_S(k_{2i}, 2k_{1f})$ dependence discussed at the end of section VIII.B of [4]. For the number N_{134}, in the cases discussed, as a lowest estimation

$$N_{134} = (V_{eff}/v_c)r_{A_1}r_{A_3}r_{A_4}, \quad (21)$$

where r_{A_i} is the relative natural abundance of isotopes of mass number A_i or $r_{A_i} = u$ if the i-th particle is a hydrogen isotope, V_{eff} is the volume effectively felt by a particle 2, u is the proton (or deuteron) over metal number density and $v_c (= d^3/4)$ is the unit cell of the solid (in the case of fcc metals).

It should be noted that there may be a great variety of other types of possible quasi-resonant heavy charged particle assisted double nuclear reactions the discussion of which is not given here.

B. Resonance-like electron assisted double nuclear processes

From the processes discussed up till now one can conclude the following: (a) it is advantageous, if the wave vector (momentum) transferred through the intermediate state by Coulomb interaction has the possible maximum value, (b) the electron assisted process is advantageous since the Coulomb and Sommerfeld factors cause minimal or negligible hindering in this case and (c) the appearance of resonance significantly increases the rate. These conclusions led us to a 4th-order, electron assisted doubled nuclear process, the graphs of which can be seen in FIG. 2. At a particular choice of the participants (see Appendix II.) resonances can be found in the processes of FIG. 2 (a) and (b). The resonance arises in line 4'. Moreover, the details of the calculation show that high contribution to the rate is obtained if the energy of the electron is negligible in the final state compared with the energy of the total reaction. In other words, the main contribution to the rate is produced by final states in which particles 6 and 7 share the reaction energy.

It is supposed that particle 1 (1' and 1") is a quasi-free electron of the solid (a metal). Particle 2 is an ingoing free, heavy, positively charged particle of system B. Particles 3, 4, and 5 are heavy particles of positive charge that are localized in the solid. If processes of FIG. 2 (a)
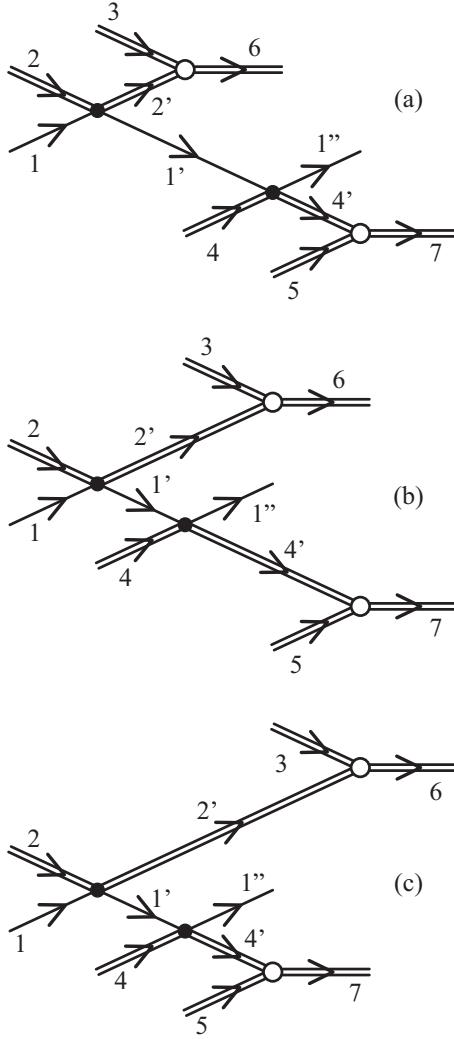
FIG. 2: The graphs of quasi-resonant, electron assisted nuclear processes. The simple lines represent free (initial (1), intermediate (1') and final (1")) electrons. The doubled lines represent heavy, charged initial (free 2, bound 4) particles (such as p, d), their intermediate states (2',4'), target nuclei (3,5) and reaction products (6,7). The filled dot denotes Coulomb-interaction and the open circle denotes nuclear (strong) interaction.

and (b) have resonance-like character (see Appendix II.) then the process of FIG. 2 (c) has not and therefore its contribution may be neglected. Now particle 4 is localized and its state is $\psi_{4i}(\mathbf{x})$ given by (4).

The total rate $W_{tot}^{(4)}$ (for the details see Appendix II.) can be obtained from the rate $W_{fi}^{(4)}$ (see (107)) as

$$W_{tot}^{(4)} = K_0^{(4)} \langle F_G \rangle \chi (\Delta) h_{corr,3}^2 h_{corr,5}^2 N_1 N_2 \frac{N_{345}}{V}, \quad (22)$$

where N_1 is the instantaneous number of the quasi-free electrons that are felt by one ingoing particle 2, N_2 is the number of quais-free ingoing particles 2, N_{345}/V is the number density of the target triples of particles 3, 4 and

5,

$$K_0^{(4)} = 2^{22} 3\pi^{23/2} \alpha_f^4 \left(\frac{f^2}{\hbar c} \right)^4 \left(1 - \frac{2}{e} \right)^4 (z_1^2 z_2 z_4)^2 (23)$$
$$\times (\hbar c)^6 R^2 c k_{1,Max}^3 \beta_4^3,$$

where $k_{1,Max}$ is the maximum of the possible wave vectors of the electron in the final state. (We calculate the rate of those processes in which the kinetic energy E_{1f} of the electron can be neglected in the energy E_f of the final state).

$$\langle F_G \rangle = \frac{\langle F_{12}(k_i) \rangle}{F_{12}(2k_4^r)} F_{23}(k_4^r) F_{45}(k_4^r), \quad (24)$$

with

$$k_4^r = \frac{\sqrt{2m_4 \delta_{a,b}}}{\hbar} = \frac{\sqrt{2m_4 \left(\Delta_6 - \frac{m_7}{m_6+m_7}\Delta \right)}}{\hbar} \quad (25)$$

Now $\Delta_6 = \Delta_{02} + \Delta_{03} - \Delta_{06}$ is the energy of reaction $2' + 3 \to 6$ and $\Delta_7 = \Delta_{05} + \Delta_{04} - \Delta_{07}$ is the energy of reaction $4' + 5 \to 7$. The Δ_{0j}-s are again the energy defects of the corresponding nuclei and the total reaction energy

$$\Delta = \Delta_6 + \Delta_7. \quad (26)$$

$$\chi (\Delta) = \frac{m_7}{m_4} \frac{\sqrt{2m_7 c^2 \Delta}}{\delta_{a,b}^3 \Delta_{ab}^2} \left(\frac{1}{\Delta_a} + \frac{1}{\Delta_b} \right)^2, \quad (27)$$

where $\delta_{a,b}$ is given by (101), furthermore, for Δ_a, Δ_b and Δ_{ab} and see (104), (105) and (106), and for $h_{corr,3}$ and $h_{corr,5}$ see (10) and (11).

It is reasonable to take

$$\frac{N_{345}}{V} = \frac{1}{v_c} r_{A_3} r_{A_4} r_{A_5} = \frac{4}{d^3} r_{A_3} r_{A_4} r_{A_5}, \quad (28)$$

where $v_c = d^3/4$ is the unit cell of the solid in the case of fcc metals, r_{A_i} is the relative natural abundance of isotopes of mass number A_i or $r_{A_i} = u$ if the i-th particle is a hydrogen isotope. Thus

$$W_{tot}^{(4)} = (\frac{4}{d^3} K_0^{(4)}) \langle F_G \rangle \chi (\Delta) h_{corr,3}^2 h_{corr,5}^2 N_1 N_2 r_{A_3} r_{A_4} r_{A_5}. \quad (29)$$

If particle 4 is a deuteron then $\beta_4 = 4.81 \times 10^8 \left[cm^{-1} \right]$. Taking $E_{1,Max} = 0.1 [MeV]$, that is the maximum of the possible energies of the electron in the final state, one gets $k_{1,Max}^3 = 4.89 \times 10^{30} \left[cm^{-3} \right]$ and

$$\frac{4}{d^3} K_0^{(4)} = 3.50 \times 10^{-3} (z_1^2 z_2 z_4)^2 \left[MeV^6 s^{-1} \right] \quad (30)$$

in the case of Pd, i.e. with $d = 3.89 \times 10^{-8} [cm]$.

As a numerical example let particles 2, 4 and 5 be deuterons, particle 3 a Pd isotope (of $A_3 = 106$ with $r_{A_3} = 0.2733$) and particle 7 $_2^4 He$. With this choice

$$\Delta_6 = 10.833 [MeV], \Delta_7 = 23.847 [MeV], \quad (31)$$

$$\Delta = 34.681 \, [MeV], \tag{32}$$

furthermore

$$\delta_{a,b} = 9.60 \, [MeV], \Delta_{ab} = 198.7 \, [MeV], \tag{33}$$

$$\Delta_a = 19.2 \, [MeV], \Delta_b = 179.5 \, [MeV], \tag{34}$$

and $m_7/m_4 = 2$ resulting

$$\chi(\Delta) = 1.69 \times 10^{-6} \, [MeV^{-6}]. \tag{35}$$

$$F_{23}(k_4^r) = 2.79 \times 10^{-8}, F_{45}(k_4^r) = 0.89, F_{12}(2k_4^r) = 1 \tag{36}$$

and

$$\langle F_{12}(k_i(E_i)) \rangle = 23.18 \times \left\langle E_i^{-1/2} \, [eV] \right\rangle_{av} \tag{37}$$

producing

$$\langle F_G \rangle = 5.76 \times 10^{-7} \times \left\langle E_i^{-1/2} \, [eV] \right\rangle_{av}, \tag{38}$$

where E_i is the energy of the initial conduction electron. Averaging $E_i^{-1/2}$ by means of the Fermi-Dirac distribution in the Sommerfeld free electron model at $T = 0$ yields

$$\left\langle E_i^{-1/2} \right\rangle_{av} = 2 \left(E_F^{-1/2} \, [eV] \right) \tag{39}$$

where E_F denotes the Fermi energy [8]. With these numbers

$$W_{tot}^{(4)} = 8.36 \times 10^{-11} \left(E_F^{-1/2} \, [eV] \right) \times N_1 N_2 u^2 \, [s^{-1}]. \tag{40}$$

This rate produces a total power $P_{tot}^{(4)} = C_E W_{tot}^{(4)} \Delta$, where $C_E = 1.602 \times 10^{-13} \, [J/MeV]$ is the energy unit conversion factor. With a deuteron concentration independent $E_F = 17 \, [eV]$,

$$P_{tot}^{(4)} = 1.13 \times 10^{-22} \times N_1 N_2 u^2 \, [W]. \tag{41}$$

In all the charged particle assisted processes discussed the quasi-resonant, 4th-order electron assisted double nuclear process seems to be the leading one.

III. NUCLEAR POWER IN NI-H SYSTEMS DUE TO DOUBLE PROTON CAPTURE

In this section we deal with a family of 4th-order resonant, electron-assisted double nuclear processes shown in FIG. 2. Our aim is to show that in $Ni-H$ systems formed in hydrogen gas one of the family of processes in FIG. 2. may have high rate and the power generated by these nuclear processes is also considerable from practical point of view that can be calculated in our theory. In $Ni-H$ systems formed in hydrogen gas extra heat production

was observed [9], [10] whose nuclear origin was proven with neutron detection [11]. Since in $Ni-H$ systems formed in H gas there is no light particle for significant nuclear effect save the natural deuteron content of hydrogen, according to our theory the primary process to generate considerable energy is the quasi-resonant electron assisted (double) proton capture of the Ni isotopes.

The two proton captures

$$_{28}^{A}Ni + p \rightarrow \, _{29}^{A+1}Cu + \Delta, \tag{42}$$
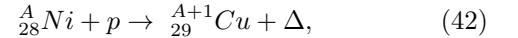
which are coupled due to the quasi-resonant electron assisted process, are investigated (see FIG. 2). Particle 1 (1' and 1") is a quasi-free electron of the metal, particle 2 is a quasi-free ingoing proton and particle 4 is a localized proton. Particles 3 and 5 are different Ni isotopes and they have mass numbers A_3 and A_5, respectively. Particles 6 and 7 are Cu isotopes of mass numbers $A_3 + 1$ and $A_5 + 1$, respectively. Both nuclear transitions $2', 3 \rightarrow 6$ and $4', 5 \rightarrow 7$ are reactions of type (42). The process is called quasi-resonant electron assisted double proton capture process. Most of the daughter nuclei $_{29}^{A+1}Cu$ decay by the

$$_{29}^{A+1}Cu + e \rightarrow \, _{28}^{A+1}Ni + Q_{EC} \tag{43}$$
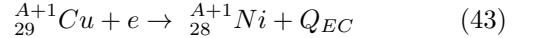
electron capture reaction. TABLE I. of [4] contains the relevant data for reactions (42) and (43). As it is discussed above at a particular choice of the participants resonances can be found in the processes of FIG. 2. The resonance appears in line 4'.

Now particle 4 is a localized proton. We take $\psi_{4i}(\mathbf{x})$ (see (4)) as initial state of particle 4. The parameter β_4 in the case of a localized proton is $\beta_4^p = \sqrt{m_p \omega_4/\hbar}$, where m_p is the proton rest mass and ω_4 is the angular frequency of the ground state of a 3-dimensional harmonic oscillator of energy $E_0 = \frac{3}{2}\hbar\omega_4$. In $NiH_{0.75}$ the energy of an optical phonon $\hbar\omega_4 = 88 \, [meV]$ [12], that results $\beta_4^p = 6.51 \times 10^8 \, [cm^{-1}]$ which is used in the calculation. The Coulomb matrix elements which contain initial state of form $\psi_{4i}(\mathbf{x})$ also preserves momentum [4]. The number density $N_{345}/V = 4d^{-3}r_{A_3}r_{A_5}u$ (see (28)), where u denotes the proton over metal number density and d is the length of the Ni elementary cell ($d = 3.52 \times 10^{-8} \, [cm]$).

Now we reformulate the results of section II. B. $\Delta_{A_3} = \Delta_{02} + \Delta_{03} - \Delta_{06}$ is the energy of reaction $2' + 3 \rightarrow 6$ and $\Delta_{A_5} = \Delta_{04} + \Delta_{05} - \Delta_{07}$ is the energy of reaction $4' + 5 \rightarrow 7$. The Δ_{0j}-s are the energy defects of the corresponding nuclei and the total reaction energy

$$\Delta_{A_3,A_5} = \Delta_{A_3} + \Delta_{A_5}. \tag{44}$$

It was found above that

$$\delta_{A_3,A_5} = \frac{(A_3 + 1)\Delta_{A_3} - (A_5 + 1)\Delta_{A_5}}{A_3 + A_5 + 2} \tag{45}$$

is a crucial quantity (it was defined by (101)), since, if $\delta_{A_3,A_5} > 0$ then resonance appears in line 4' at

$$k_{4;A_3,A_5}^r = \frac{\sqrt{2m_4 \delta_{A_3,A_5}}}{\hbar}. \tag{46}$$

Substituting

$$h_{corr,3}^2 h_{corr,5}^2 = (A_3 - 28)^2 (A_5 - 28)^2 \quad (47)$$

into (22) the total rate of the 4th-order, resonance-like electron assisted double proton capture processes has the form

$$W_{tot}^{(4)}(A_3, A_5) = \frac{4}{d^3} K_0^{(4)} (A_3 - 28)^2 (A_5 - 28)^2 \quad (48)$$
$$\times \langle F_G \rangle \chi N_1 N_2 r_{A_3} r_{A_5} u.$$

Here N_1 is the instantaneous number of the quasi-free electrons that are felt by an ingoing particle 2, N_2 is the number of quasi-free ingoing protons that can interact with particle triplets 3, 4 and 5, and r_{A_3} and r_{A_5} are the relative natural abundances of Ni isotopes of mass numbers A_3 and A_5, respectively (see TABLE I. of [4]). $K_0^{(4)}$ given by (23), where $R = 1.2 \times 10^{-13}\,[cm]$ is the proton-radius. Taking $E_{1,Max} = 0.1\,[MeV]$ again one gets

$$4d^{-3} K_0^{(4)} = 0.0118 \left[MeV^6 s^{-1} \right] \quad (49)$$

in the case of Ni. Since $\mu_{23} \simeq \mu_{45}$ and $z_2 z_3 = z_4 z_5 = 28$ therefore $F_{23}(k_{4;A_3,A_5}^r) \simeq F_{45}(k_{4;A_3,A_5}^r)$ in $\langle F_G \rangle$. Furthermore, since particle 1 is an electron therefore $F_{12}(2k_{4;A_1,A_2}^r) = 1$ and (24) reads

$$\langle F_{G;A_3,A_5} \rangle = \langle F_{12}(k_i) \rangle F_{23}^2 (k_{4;A_3,A_5}^r). \quad (50)$$

Here $\langle F_{12}(k_i(E_i)) \rangle = 46.36 \times \left(E_F^{-1/2}\,[eV] \right)$, where E_F denotes the Fermi energy. If $2m_p c^2 \gg \delta$, true in our case, then (27) can be written as

$$\chi_{A_3,A_5} = \frac{(A_5+1)^{3/2} \sqrt{2m_0 c^2 \Delta_{A_3,A_5}}}{4\delta_{A_3,A_5}^7}. \quad (51)$$

Here $m_0 c^2 = 931.494\,[MeV]$ is the atomic mass unit.

One can see from TABLE I. of [4] that Δ_A increases monotonically with the increase of A. Thus $\delta_{A_3,A_5} > 0$ if $A_3 > A_5$, and the total power can be written as

$$P_{tot} = N_1 N_2 u P_0 \sum_{A_3 > A_5} \psi_{A_3,A_5}, \quad (52)$$

where $P_0 = 4d^{-3} K_0^{(4)} C_E \Delta_0^{-5}$. $C_E = 1.602 \times 10^{-13}$ $[J/MeV]$ is the energy unit conversion factor, $\Delta_0 = 1$ $[MeV]$ as an order of magnitude of a typical nuclear reaction energy value. The use of Δ_0 makes

$$\psi_{A_3,A_5} = (A_3 - 28)^2 (A_5 - 28)^2 r_{A_3} r_{A_5} \times \quad (53)$$
$$\times \Delta_{A_3,A_5} \langle F_{G;A_3,A_5} \rangle \chi_{A_3,A_5} \Delta_0^5$$

dimensionless. In our case

$$P_0 = 1.88 \times 10^{-15}\,[W] \quad (54)$$

and

$$\sum_{A_3 > A_5} \psi_{A_3,A_5} = 6.48 \times 10^{-6} \times \left(E_F^{-1/2}\,[eV] \right). \quad (55)$$

In $\sum_{A_3 > A_5} \psi_{A_3,A_5}$ the significant contributions are given by $\psi_{61,58}$, $\psi_{62,58}$, $\psi_{64,58}$ and $\psi_{64,60}$, with $\psi_{64,58} = 6.36 \times 10^{-6} \times \left(E_F^{-1/2}\,[eV] \right)$ as the leading term responsible for 98% of the effect. A hydrogen concentration independent $E_F = 17\,[eV]$ is used producing

$$P_{tot} = 2.95 \times 10^{-21} \times N_1 N_2 u\,[W]. \quad (56)$$

Now we proceed to the determination of N_1, N_2 and u. N_1 stands for the number of valence electrons which can interact with a proton (particle 2). Considering that Ni consists of micro crystals of linear dimension of about $(D\,[\mu m])$ $1 - 10$ μm, it is reasonable to assume that a proton penetrating the material "feels" all the valence electrons. Thus

$$N_1 = \frac{V_g}{v_c} g_e, \quad (57)$$

where $V_g = D^3 \times 10^{-12}\,[cm^3]$ is the volume of micro crystals, $v_c = d^3/4$ is the volume of the elementary cell and $g_e = 10$ is the number of valence electrons in an elementary cell of Ni. From it $N_1 = D^3\,[\mu m] \times 9.17 \times 10^{11}$. N_2 denotes the number of protons which can be considered quasi free in respect of the process. It can be calculated if the number density n_p in gas is multiplied with the metal volume V_2, where they are taken for free. The volume is the product of the surface F of the sample and the length d of the elementary cell. The result is

$$N_2 = 2F n_L \frac{p T_0}{p_0 T} d, \quad (58)$$

where $n_L = 2.69 \times 10^{19}\,[cm^{-3}]$ is the number density in the gas of pressure $p_0 = 1\,[atm]$ and temperature $T_0 = 273\,[^0K]$. The actual pressure and temperature are denoted by p and T. Factor 2 follows from the fact that the hydrogen molecule contains 2 atoms. Here the catalytic process producing atomic hydrogen is not considered, it supposed, which is a rough over estimation, that at the Ni surface the whole gas is atomistic.

In [9] excess heat power $44\,[W]$ is obtained from a Ni rod of diameter $d_0 = 0.5\,[cm]$ and length $h_0 = 9\,[cm]$ at $p = 0.5\,[atm]$ and $T = 753\,[^0K]$. At this temperature and pressure $u \simeq 7.9 \times 10^{-5}$ [13]. Utilizing this and the values N_1 and N_2 obtained above, $P_{tot} = D^3 \times 0.074$ $[W]$ from (56). In case of $D \simeq 8.41\,[\mu m]$ the result is $44\,[W]$, which considering that in this simple model a great number of solid state processes were neglected and in the nuclear processes it was only the Weisskopf approximation in which the matrix element was calculated, is a very good approximation.

The surface area of the sample, as can be seen from (58), can significantly increase the rate of the processes

in the sample and the power generated. For example, if we take a sample of similar volume to the previous one, but not in the shape of a rod but powder of particles of diameter D_p then the power is increased by a factor of $3d_0/(2D_p)$. If $D_p = 10^{-3}\,[cm]$ then this factor is 750, that is a sample of similar volume can produce $33\,[kW]$ instead of $44\,[W]$. It seems that this is what [14] found experimentally who applies Ni powder in his patented device. All this means that the above mechanism can produce power generation for practical applications.

IV. SUMMARY

Resonance-like heavy particle and electron assisted double nuclear processes in solids are discussed. The transition probabilities per unit time of the 3rd-order heavy particle assisted and the 4th-order electron assisted resonance-like double nuclear processes are determined. The 3rd-order heavy particle assisted and the 4th-order electron assisted resonance-like double nuclear processes may partly be responsible for the so called anomalous screening effect observed in low energy accelerator physics investigating astrophysical factors of nuclear reactions of low atomic numbers [15]. The theoretical description of the doubled processes discussed extends the possible explanation and description of LENR with its nuclear physical background. It is found that the $d + d \rightarrow {}^4He$ process coupled to the ${}^A_{46}Pd + d \rightarrow {}^{A+1}_{47}Ag$ process due to the quasi-resonant electron assisted doubled nuclear process has extremely large rate. The ${}^4_2He$ production with $34.7\ MeV/He$ obtained in the leading, 4th-order, quasi-resonant electron assisted $d + d \rightarrow {}^4_2He$ process fits well with the observed $32\pm 13\ MeV/He$ value of LENRs [3].

With the help of our theory describing resonance-like electron assisted doubled nuclear processes we estimated the nuclear power (41) created in the $d + d \rightarrow {}^4He$ process coupled to the ${}^A_{46}Pd + d \rightarrow {}^{A+1}_{47}Ag$ process due to the quasi-resonant electron assisted doubled nuclear process and the nuclear power (56) due to double proton capture in $Ni - H$ systems formed by Ni placed in H_2 gas environment. The nuclear powers are consistent with observations. Moreover, the magnitude of the power obtained in $Ni - H$ systems containing Ni in powdered form is considerable from practical point of view.

V. APPENDIX I. RATE OF RESONANCE-LIKE HEAVY CHARGED PARTICLE ASSISTED DOUBLED NUCLEAR PROCESSES

The rate of the process is

$$W_{fi}^{(3)} = \frac{2\pi}{\hbar} \sum_f \left| T_{if}^{(3)} \right|^2 \delta(E_f - \Delta) \qquad (59)$$

where

$$T_{if}^{(3)} = \sum_\nu \sum_\mu \frac{V_{f\nu}^{St} V_{\nu\mu}^{St} V_{\mu i}^{Cb}}{(E_\nu - E_i)(E_\mu - E_i)} \qquad (60)$$
$$\times \frac{(2\pi)^3}{V} \delta(\mathbf{k}_5 + \mathbf{k}_6).$$

$\mathbf{k}_j$ denotes the wave vector of particle j in the final state. It will be seen that the process may have resonance like character if the masses of particles 5 and 6 differ significantly, therefore the $T_{if}^{(3)}$ attached to the leading graph (e.g. FIG. 1(a) in the case $m_5 \gg m_6$ discussed) is enough to calculate. $\mathbf{k}_{jn}$ denotes the wave vector of particle j in state $n = i, \mu$ or ν. The initial wave vector $\mathbf{k}_{2i}$ of particle 2 is neglected in the Coulomb matrix element

$$V_{\mu i}^{Cb} = g_S \frac{2(2\pi)^4 e^2 z_1 z_2}{V^{3/2}} \frac{\widetilde{\psi}_{1i}(\mathbf{k}_{1\mu} + \mathbf{k}_{2\mu})}{|\mathbf{k}_{2\mu}|^2 + \lambda^2}, \qquad (61)$$

where the

$$\widetilde{\psi}_{1i}(\mathbf{k}_{1\mu} + \mathbf{k}_{2\mu}) = 8\pi^{9/4} \beta_1^{3/2} \delta(\mathbf{k}_{1\mu} + \mathbf{k}_{2\mu}) \qquad (62)$$

formula can be used assuming that particle 1 is localized. $\widetilde{\psi}_{1i}$ stands for the Fourier transform of the initial state ψ_{1i}.

The two nuclear matrix elements

$$V_{\nu\mu}^{St,W}(a) = -2f^2 f_{14}(k_{1\mu}) \sqrt{\frac{12\pi R}{V}} \left(1 - \frac{2}{e}\right) \times \qquad (63)$$
$$\times \frac{(2\pi)^3}{V} \delta(\mathbf{k}_{1\mu} - \mathbf{k}_{1\nu}),$$

and

$$V_{f\nu}^{St,W}(a) = -2f^2 f_{23}(k_{2\nu}) \sqrt{\frac{12\pi R}{V}} \left(1 - \frac{2}{e}\right) \qquad (64)$$

are valid in the case of the process of FIG. 1(a) and

$$V_{\nu\mu}^{St,W}(b) = -2f^2 f_{23}(k_{2\mu}) \sqrt{\frac{12\pi R}{V}} \left(1 - \frac{2}{e}\right) \qquad (65)$$
$$\times \frac{(2\pi)^3}{V} \delta(\mathbf{k}_{1\mu} - \mathbf{k}_{1\nu}),$$

$$V_{f\nu}^{St,W}(b) = -2f^2 f_{14}(k_{1\nu}) \sqrt{\frac{12\pi R}{V}} \left(1 - \frac{2}{e}\right) \qquad (66)$$

stand for the process of FIG. 1(b). Here R is the radius of a nucleon (we take $R = 1.2 \times 10^{-13}\,[cm]$, that is the proton-radius) and the single nucleon approach in the Weisskopf approximation is used. The energy differences in the denominator of (60) are: the difference of the kinetic energies

$$E_\mu - E_i = \frac{\hbar^2 k_{1\mu}^2}{2m_1} + \frac{\hbar^2 k_{2\mu}^2}{2m_2} - E_{i,kin} \qquad (67)$$

and

$$(E_\nu - E_i)_a = \frac{\hbar^2 k_{2\nu}^2}{2m_2} + \frac{\hbar^2 k_5^2}{2m_5} - E_{i,kin} - \Delta_5 \qquad (68)$$

in the case of FIG. 1(a) and

$$(E_\nu - E_i)_b = \frac{\hbar^2 k_{1\nu}^2}{2m_1} + \frac{\hbar^2 k_6^2}{2m_6} - E_{i,kin} - \Delta_6 \qquad (69)$$

in the case of FIG. 1(b). In (68) and (69) the rest energy differences have to be also taken into account because of the nuclear reaction $1' + 4 \to 5$ in the case of (a) and $2' + 3 \to 6$ in the case of (b). $\Delta_5 = \Delta_{01} + \Delta_{04} - \Delta_{05}$ is the energy of reaction $1' + 4 \to 5$, and $\Delta_6 = \Delta_{02} + \Delta_{03} - \Delta_{06}$ is the energy of reaction $2' + 3 \to 6$. The Δ_{0j}-s are the energy defects of the corresponding nuclei. The total reaction energy

$$\Delta = \Delta_5 + \Delta_6. \qquad (70)$$

After performing the $\sum_f$ (really using the $\sum_f \to \int \left[V/(2\pi)^3 \right] d\mathbf{k}_5 \times \int \left[V/(2\pi)^3 \right] d\mathbf{k}_6$ correspondence and integrating over $\mathbf{k}_6$) in (59) the Dirac delta $\delta(\mathbf{k}_5 + \mathbf{k}_6)$ in (60) will result $\mathbf{k}_6 = -\mathbf{k}_5$, and the final energy can be written as $E_f = \hbar^2 k_5^2 / (2\mu_{56})$. The energy Dirac delta will result

$$k_5^2 = \frac{2\mu_{56}\Delta}{\hbar^2}. \qquad (71)$$

Furthermore because of the presence of $\delta(\mathbf{k}_{2\mu} - \mathbf{k}_{2\nu})$ and $\delta(\mathbf{k}_{1\mu} - \mathbf{k}_{1\nu})$ in the matrix elements $V_{\nu\mu}^{St,W}(a)$ and $V_{\nu\mu}^{St,W}(b)$ the $\sum_\nu \to \int \left[V/(2\pi)^3 \right] d\mathbf{k}_{2\nu}$ and $\sum_\nu \to \int \left[V/(2\pi)^3 \right] d\mathbf{k}_{1\nu}$ will allow the $\mathbf{k}_{2\nu} = \mathbf{k}_{2\mu}$ and $\mathbf{k}_{1\nu} = \mathbf{k}_{1\mu}$ substitutions in them and in the energy denominators as well. The initial kinetic energy $E_{i,kin}$ is neglected in $(E_\nu - E_i)_a$ and $(E_\nu - E_i)_b$. Thus

$$(E_\nu - E_i)_a = \frac{\hbar^2 k_{2\mu}^2}{2m_2} + \delta_a \qquad (72)$$

with

$$\delta_a = \frac{\mu_{56}}{m_5}\Delta - \Delta_5 \qquad (73)$$

and

$$(E_\nu - E_i)_b = \frac{\hbar^2 k_{1\mu}^2}{2m_1} + \delta_b \qquad (74)$$

with

$$\delta_b = \frac{\mu_{56}}{m_6}\Delta - \Delta_6. \qquad (75)$$

Since

$$\delta_a + \delta_b = 0 \qquad (76)$$

one of δ_a and δ_b is negative. Let us suppose that $\delta_a < 0$. It means if

$$\frac{\hbar^2 k_{2\mu}^2}{2m_2} = -\delta_a = \Delta_5 - \frac{\mu_{56}}{m_5}\Delta \qquad (77)$$

then $(E_\nu - E_i)_a = 0$ and we find that the process (a) has resonance-like behavior when $k_{2\mu} = k^r$ with $k^r = \sqrt{2m_2\left(\Delta_5 - \frac{\mu_{56}}{m_5}\Delta\right)}/\hbar$. If $\delta_a < 0$ then $\delta_b > 0$ and the process (b) can not have resonance character, and therefore it is enough to calculate the $T_{if}^{(3)}$ attached to graph 3(a).

Let us introduce the half width Γ of the resonance with which the complex energy difference, denoted by suffix C, reads

$$(E_\nu - E_i)_{a,C} = \frac{\hbar^2 k_{2\mu}^2}{2m_2} + \delta_a - i\frac{\Gamma}{2} \qquad (78)$$

that equals $-i\frac{\Gamma}{2}$ if the resonance condition (77) is met. With the use of the correspondence $\sum_\mu \to \int \left[V/(2\pi)^3 \right] d\mathbf{k}_{2\mu} \times \int \left[V/(2\pi)^3 \right] d\mathbf{k}_{1\mu}$ in (60) and after carrying out the integral over $\mathbf{k}_{1\mu}$ the relation (62) gives $\mathbf{k}_{1\mu} = -\mathbf{k}_{2\mu}$. Integrating over $\mathbf{k}_{2\mu}$ we have the integral of the form

$$I = \int \frac{h(k_{2\mu})\, d\mathbf{k}_{2\mu}}{\frac{\hbar^2 k_{2\mu}^2}{2m_2} + \delta_a - i\frac{\Gamma}{2}}, \qquad (79)$$

where $h(k_{2\mu})$ is any function of $k_{2\mu}$, that in this case

$$h(k_{2\mu}) = g_S(k_{2i}, 2k_{2\mu}) f_{23}(k_{2\mu}) f_{14}(k_{2\mu}). \qquad (80)$$

For evaluating (79) the $g(k_{2\mu}) = \frac{\hbar^2 k_{2\mu}^2}{2m_2} + \delta_a$ is introduced and the substitution

$$\frac{1}{g(k_{2\mu}) - i\varepsilon} \to P\frac{1}{g(k_{2\mu})} + i\pi\delta(g(k_{2\mu})) \qquad (81)$$

is applied resulting

$$I = I_P + iI_\delta, \qquad (82)$$

where

$$I_P = \int P\frac{h(k_{2\mu})\, d\mathbf{k}_{2\mu}}{\frac{\hbar^2 k_{2\mu}^2}{2m_2} + \delta_a} \qquad (83)$$

and

$$I_\delta = \pi \int h(k_{2\mu})\, \delta(g(k_{2\mu}))\, d\mathbf{k}_{2\mu}. \qquad (84)$$

Using the identity

$$\delta(g(k_{2\mu})) = \frac{\delta(k_{2\mu} - k^r)}{\frac{d}{dk_{2\mu}} g(k_{2\mu})\, |_{k_{2\mu}=k^r}} \qquad (85)$$

where k^r (see (9)) is the root of the $g(k_{2\mu}) = 0$ equation (see (77)). In a lower estimation of I as $|I| = I_\delta$, the relevant part of (60) is approximated as

$$\left| \int \frac{h(k_{2\mu})}{\frac{\hbar^2 k_{2\mu}^2}{2m_2} + \delta_a - i\frac{\Gamma}{2}} d\mathbf{k}_{2\mu} \right| \qquad (86)$$
$$= 4\pi^2 k^r m_2 h(k^r) / \hbar^2.$$

Applying the above relations and results, a lower approximation of the transition rate reads as

$$W_{fi}^{(3)} = K^{(3)} K_1^{(3)} (\Delta, \Delta_5) H(k^r) h_{corr,3}^2 h_{corr,4}^2 \qquad (87)$$

where $K^{(3)}$, $K_1^{(3)}(\Delta, \Delta_5)$ and $H(k^r)$ are given by (6), (7) and (8), and for $h_{corr,3}$ and $h_{corr,5}$ see (10) and (11), that are defined by Eq.(45) of [4], and k^r is determined by (9).

VI.   APPENDIX II. RATE OF RESONANCE-LIKE ELECTRON ASSISTED DOUBLED NUCLEAR PROCESSES

The rate of the process

$$W_{fi}^{(4)} = \frac{2\pi}{\hbar} \sum_f \left| T_{if}^{(4)} \right|^2 \delta(E_f - \Delta) \qquad (88)$$

with

$$T_{if}^{(4)} = T_{if}^{(4)}(a) + T_{if}^{(4)}(b) + T_{if}^{(4)}(c), \qquad (89)$$

where $T_{if}^{(4)}(a)$, $T_{if}^{(4)}(b)$ and $T_{if}^{(4)}(c)$ are the matrix elements of the processes of FIG. 2 (a), (b) and (c), respectively. If processes of FIG. 2 (a) and (b) have resonance-like character (see later) then the process of FIG. 2 (c) has not and therefore its contribution may be neglected. Thus we take into account the contributions of

$$T_{if}^{(4)}(a) = \sum_\rho \sum_\nu \sum_\mu \frac{V_{f\rho}^{St} V_{\rho\nu}^{Cb} V_{\nu\mu}^{St}}{(E_\rho - E_i)(E_\nu - E_i)} \qquad (90)$$
$$\times \frac{V_{\mu i}^{Cb}}{(E_\mu - E_i)} \frac{(2\pi)^3}{V} \delta(\mathbf{k}_{1f} + \mathbf{k}_6 + \mathbf{k}_7)$$

and

$$T_{if}^{(4)}(b) = \sum_\rho \sum_\nu \sum_\mu \frac{V_{f\rho}^{St} V_{\rho\nu}^{St} V_{\nu\mu}^{Cb}}{(E_\rho - E_i)(E_\nu - E_i)} \qquad (91)$$
$$\times \frac{V_{\mu i}^{Cb}}{(E_\mu - E_i)} \frac{(2\pi)^3}{V} \delta(\mathbf{k}_{1f} + \mathbf{k}_6 + \mathbf{k}_7).$$

The outline of the calculation is the following. For the Coulomb matrix element of the process $1, 2 \to 1', 2'$ we use the form $V_{\mu i}^{Cb}$ given by Eq.(37) of [4]. For calculating the matrix element of the Coulomb interaction of the process $1', 4 \to 1'', 4'$ the form given by (61) and the

approximation (62) are used. Thus in each filled dot of the graphs representing a Coulomb interaction the momentum (wave number) is conserved. The initial wave vectors of particles 1, 2 and 4 are neglected. The matrix elements of nuclear transitions $2', 3 \to 6$ and $4', 5 \to 7$ are calculated in the Weisskopf approximation applying the appropriate one from formulae (63), (65), (64) and (66) with the appropriate $f_{23}(k_2)$ and $f_{45}(k_4)$ functions in it, respectively. The initial motion of particles 3 and 5, i.e. their initial wave vectors are also neglected. In summing up for the intermediate and final states and for the square of the Dirac delta of argument of wave vector the correspondences and relations used above are applied again. (Remember, that now $\Delta_6 = \Delta_{02} + \Delta_{03} - \Delta_{06}$ is the energy of reaction $2' + 3 \to 6$ and $\Delta_7 = \Delta_{05} + \Delta_{04} - \Delta_{07}$ is the energy of reaction $4' + 5 \to 7$. The Δ_{0j}-s are again the energy defects of the corresponding nuclei and the total reaction energy $\Delta = \Delta_6 + \Delta_7$). We calculate the rate of those processes in which the kinetic energy E_{1f} of the electron can be neglected in the energy E_f of the final state. Consequently the $V/(2\pi)^3 \int d\mathbf{k}_{1f}$ will result a factor $V/(2\pi)^3 (4\pi/3) k_{1,Max}^3$, where $k_{1,Max}$ is the maximum of the possible wave vectors of the electron in the final state. Furthermore, in the energy Dirac delta

$$E_f = E_6 + E_7 - \Delta \qquad (92)$$

is used. Neglecting also the final wave vector $\mathbf{k}_{1f}$ of the electron in the Dirac delta, $\delta(\mathbf{k}_6 + \mathbf{k}_7)$ is used in (90), (91) resulting

$$E_f = \frac{\hbar^2 k_7^2}{2\mu_{67}} - \Delta \qquad (93)$$

after integration over $\mathbf{k}_6$ ($k_6 = k_7$).

Since in the cases investigated $\mathbf{k}_{1f}$ is neglected, the wave number vector conservation in Coulomb scattering results the conservation of the magnitude of wave vector in lines 1', 2' and 4', and it is denoted by k_4. Let us now investigate the energy denominators. The intermediate states are labeled with μ, ν and ρ. In the case of graph (a)

$$(E_\mu - E_i)_a = \frac{\hbar^2 k_4^2}{2m_2} + \sqrt{(\hbar c k_4)^2 + m_e^2 c^4} - m_e c^2, \quad (94)$$

$$(E_\nu - E_i)_a = \frac{\hbar^2 k_7^2}{2m_6} - \Delta_6 + \sqrt{(\hbar c k_4)^2 + m_e^2 c^4} - m_e c^2, \qquad (95)$$

and

$$(E_\rho - E_i)_a = \frac{\hbar^2 k_7^2}{2m_6} - \Delta_6 + \frac{\hbar^2 k_4^2}{2m_4}. \qquad (96)$$

In obtaining (96) E_{1f} is neglected. In the case of graph (b)

$$(E_\mu - E_i)_b = (E_\mu - E_i)_a, \qquad (97)$$

$$(E_\nu - E_i)_b = \frac{\hbar^2 k_4^2}{2m_2} + \frac{\hbar^2 k_4^2}{2m_4}, \qquad (98)$$

and

$$(E_\rho - E_i)_b = \frac{\hbar^2 k_7^2}{2m_6} - \Delta_6 + \frac{\hbar^2 k_4^2}{2m_4}. \qquad (99)$$

One can see from (96) and (99) that $(E_\rho - E_i)_a = (E_\rho - E_i)_b$. Integrating over $\mathbf{k}_7$ and using the energy Dirac delta

$$(E_\rho - E_i)_a = (E_\rho - E_i)_b = \frac{\hbar^2 k_4^2}{2m_4} - \delta_{a,b} \qquad (100)$$

with

$$\delta_{a,b} = \Delta_6 - \frac{\mu_{67}}{m_6}\Delta = \Delta_6 - \frac{m_7}{m_6 + m_7}\Delta. \qquad (101)$$

If $\delta_{a,b} > 0$ then in (90) and (91) resonance appears at k_4^r given by (25). Let us introduce again the half width Γ of the resonance with which the complex energy differences read

$$(E_\rho - E_i)_{a,C} = (E_\rho - E_i)_{b,C} = \frac{\hbar^2 k_4^2}{2m_4} - \delta_{a,b} - i\frac{\Gamma}{2}. \quad (102)$$

The integration over $\mathbf{k}_4$

$$\left| \int \frac{f_{23}(k_4)f_{45}(k_4)}{\frac{\hbar^2 k_4^2}{2m_4} - \delta_{a,b} - i\frac{\Gamma}{2}} d\mathbf{k}_4 \right| \qquad (103)$$
$$= 4\pi^2 k_4^r m_4 f_{23}(k_4^r)f_{14}(k_4^r)/\hbar^2,$$

with the aid of (86), where k_4^r is given by (25). (Now, because particle 1 is an electron $g_S(k_{11'}, k_{4'1'}) \simeq g_S(k_{1'}, k_{4'}) = 1$.)

The energy differences in the denominators of (90) and (91) will be

$$(E_\nu - E_i)_a = \delta_{a,b}\left(\sqrt{\frac{2m_4 c^2}{\delta_{a,b}}} - 1\right) = \Delta_a, \qquad (104)$$

$$(E_\nu - E_i)_b = \frac{m_4}{\mu_{24}}\delta_{a,b} = \Delta_b, \qquad (105)$$

and

$$(E_\mu - E_i)_b = (E_\mu - E_i)_a = \delta_{a,b}\left(\frac{m_4}{m_2} + \sqrt{\frac{2m_4 c^2}{\delta_{a,b}}}\right) = \Delta_{ab}. \qquad (106)$$

Now the rate of the 4th-order processes is

$$W_{fi}^{(4)} = K_0^{(4)} F_G \frac{\chi(\Delta)}{V} h_{corr,3}^2 h_{corr,5}^2 \qquad (107)$$

where $K_0^{(4)}$, F_G and $\chi(\Delta)$ are determined by (23), (24) and (27), k_4^r is given by (25) and for $h_{corr,3}$ and $h_{corr,5}$ see (10) and (11).

[1] M. Fleishmann and S. Pons, J. Electroanal. Chem. **261**, 301-308 (1989).

[2] S. B. Krivit and J. Marwan, J. Environ. Monit. **11**, 1731-1746 (2009).

[3] E. Storms, Naturwissenschaften, **97**, 861-881 (2010).

[4] P. Kálmán and T. Keszthelyi, arXiv:1303.1078v1 [nucl-th].

[5] J. D. Bjorken and S. D. Drell, *Relativistic Quantum Mechanics* (McGraw-Hill, New York, 1964).

[6] W. Heitler, *The Quantum Theory of Radiation*, 3rd ed. (Clarendon, Oxford, 1954) Ch. V.,25§, (19).

[7] C. Cohen-Tannoudji, B. Diu, and F. Laloë, *Quantum Mechanics*, Vol. 1. (Wiley, New York/English version/, Hermann, Paris, 1977).

[8] J. Sólyom, *Fundamentals of the Physics of Solids*, Vol.II., *Electronic Properties* (Springer, Berlin-Heidelberg, 2009), pp.35-37.

[9] S. Focardi, R. Habel, and F. Piantelli, Nuovo Cimento **107**, 163-167 (1994).

[10] S. Focardi, V. Garbani, V. Montalbano, F. Piantelli, and S. Veronesi, Nuovo Cimento **111**, 1233-1242 (1998).

[11] A. Battaglia *et al.*, Nuovo Cimento **112**, 921-931 (1999).

[12] J. Eckert, C. F. Majkzrak, L. Passell, and W. B. Daniels, Phys. Rev. B **29**, 3700-3702 (1984).

[13] J. D. Fast: *Interaction of Metals and Gases*, Vol. 1. *Thermodynamics and Phase Relations* (Philips Technical Library, 1965) p.68.

[14] A. Rossi, *Method and Apparatus for Carrying out Nickel and Hydrogen Exothermal Reactions*, patent, International Publication Number: WO 2009/125444 A1, 15 October 2009.

[15] F. Raiola *et al.*, Eur. Phys. J. A **13**, 377-382 (2002); Phys. Lett. B **547**, 193-199 (2002); C. Bonomo *et al.*, Nucl. Phys. A**719**, 37c-42c (2003); J. Kasagi *et al.*, J. Phys. Soc. Japan, **71**, 2881-2885 (2002); K. Czerski *et al.*, Europhys. Lett. **54**, 449-455 (2001); Nucl. Instr. and Meth. B **193**, 183-187 (2002); A. Huke, K. Czerski and P. Heide, Nucl. Phys. A**719**, 279c-282c (2003); A. Huke *et al.*, Phys. Rev. C **78**, 015803 (2008).