

Study of the Influence of Remote Undamped Temperature Waves on Nuclear Fusion

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Abstract—The results of studying the influence of undamped thermal waves on the process of nuclear fusion in a remote target are presented. These waves form on the reverse side of a metal target, which is subjected to a water jet in the cavitation state, and are characterized by strictly defined frequencies (in air, under normal conditions, and at different humidities, the minimum frequency of such a wave is $\omega_0 \approx 75\text{--}85$ MHz). It is shown that, the action of such waves on a remote sample of deuterated polycrystalline titanium (a “nuclear” target) generates alpha particles, the emission direction of which agrees with the target geometry and orientation.

Keywords: heat conduction equation, thermal (temperature) waves, cavitation, low-energy nuclear fusion, alpha particles

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INTRODUCTION

The authors of [1, 2] theoretically predicted the possibility of the existence of undamped temperature waves, which can propagate without dissipation over a large distance (this was confirmed experimentally in [3–6]). Such waves can exist in various media only at definite discrete frequencies that are characteristic for these media $\omega_n = (n + 1/2)\pi/\tau$, $n = 0, 1, 2, \dots$. These frequencies are determined only by the duration τ of the local thermodynamic relaxation in a given medium. The physical mechanism of the existence of such waves is due to the possibility of local thermal-energy transfer reversibility within the limits of a time instant that is less than the time of this relaxation. The authors of [3–6] also showed theoretically that, in air, the minimum frequency of such waves depends on its humidity, the pressure and the temperature; later, it was confirmed in experiments. Under normal conditions, it is $\omega_0 \approx 75\text{--}85$ MHz. In our preceding papers, we studied the process of the generation and propagation of such waves but did not analyze their influence on nuclear processes occurring in various media.

In this paper, we present results of the influence of such waves on the probability of nuclear reactions in solid-state targets at a large distance. We study the possibility of stimulating a nuclear-fusion reaction during the controlled remote pulsed action of such a wave on a polycrystalline solid target (deuterated titanium) containing deuterium at room temperature.

CONDITIONS FOR THE EXISTENCE AND FEATURES OF THE PROPAGATION OF UNDAMPED TEMPERATURE WAVES

In some of our experimental papers where cavitation phenomena in a water jet were studied [3–6], we discovered previously unknown physical processes related to the recording of high-frequency oscillations by means of an acoustic detector at a large distance from the external surface of a cavitation chamber. These oscillations cannot be identified as hypersound because, in air, hypersound waves with a frequency of $\omega_0 \geq 70\text{--}250$ MHz do not propagate and damp at a distance of several microns. Detailed analysis and comparison with previously carried-out theoretical calculations [1, 2] showed that the recorded signals corresponded to a special and previously unknown type of thermal (temperature) waves rather than to acoustic (hypersound) waves. These waves differ in terms of anomalously small absorption (and in terms of zero absorption at certain frequencies).

The possibility of the existence of such waves follows from certain features of classical thermodynamics, which were not considered previously.

One of the basic equations for heat transfer is the phenomenological Fourier law, which determines the temperature dependence of the heat flux $\vec{q}(\vec{r}, t)$ and has the form

$$\vec{q}(\vec{r}, t) = -\lambda \text{grad}(T(\vec{r}, t)), \quad (1)$$

in a material medium with a constant heat-conduction coefficient λ .

If this equation is combined with the energy conservation law for a local region (the continuity equation) in a medium with volume (mass) density ρ and the heat capacity c_v :

$$\rho c_v \frac{\partial T(\vec{r}, t)}{\partial t} = \text{div} \vec{q}(\vec{r}, t), \quad (2)$$

then it is possible to obtain the classical thermal-diffusivity equation without distributed sources in the medium:

$$\rho c_v \frac{\partial T(\vec{r}, t)}{\partial t} = \lambda \text{div} \{ \text{grad}[T(\vec{r}, t)] \}, \quad (3)$$

describing the propagation of temperature waves in a specific medium.

The general solution of the one-dimensional variant of this equation has the well-known form

$$T = A \exp(-\kappa x) \exp\{i(\omega t - \kappa x)\} + B \exp(\kappa x) \exp\{i(\omega t + \kappa x)\}. \quad (4)$$

Here, $\kappa = \sqrt{\omega/2G}$, $G = \lambda/\rho c_v$ is the thermal-diffusivity coefficient, which depends on the heat-conduction coefficient, the density, and the heat capacity of the medium.

It is seen from this solution that the damping coefficient $\delta = \kappa$ and the wave number $k = \kappa$ of such a thermal wave coincide. Such a result corresponds to the case where the thermal wave damps at a distance of several space periods; in this case, the region of wave existence decreases continuously with increasing frequency. It should be noted that, because of such extremely strong wave attenuation, such waves are not interesting for some practical applications and are mentioned more frequently as an insignificant physical fact (for example, the existence of thermal waves coming from a surface and caused by annual temperature variations in the soil).

Detailed analysis shows that the resultant relation $\delta = k$ is not universal and can be violated significantly under certain conditions. This is related to the implicit use of the phenomenological principle of *local thermodynamic equilibrium* underlying classical thermodynamics when deriving the above-mentioned relations. This principle postulates the possibility of describing a nonequilibrium system in which gradients of the temperature, concentration, and so on are present via local equilibrium states of small subsystems. In this case, the entire macrosystem is divided conditionally into small subsystems, each of which is always equilibrium inside, and all nonequilibrium processes occur by means of the interaction of these subsystems. It is quite obvious that, within the limits of these subsystems, such an assumption of local equilibrium is valid only in the case of slow processes where the actual relaxation time of each of the subsystems to the equi-

librium state τ is much smaller than the characteristic time of the nonstationary process under study (for example, the duration of the thermal front for the pulsed process of heat excitation or the period for the periodic one).

In a gas, the relaxation time (the time of local thermalization or “maxwellization”) $\tau \approx 10/n\langle\sigma(v)v\rangle$ is determined by the elastic scattering cross section $\sigma(v)$, the current velocity v of medium particles with the inclusion of their heating by a thermal wave, and by the concentration of these particles n . As the air temperature and density or composition change (for example, in the presence of water vapor), the quantity τ can vary within broad limits ($\tau \approx 10^{-7} - 10^{-8}$ s). In water, $\tau \approx 1 - 10$ ps. In metals and semiconductors, $\tau \approx 10^{-13} - 10^{-10}$ s. In plasma, within the limits of the subsystem of electrons, in a small region, the formation time of the equilibrium Maxwell distribution is:

$$\tau^{(ee)} \approx \sqrt{m_e} (k_B T_e)^{3/2} / 4\pi \Lambda n_e e^4, \quad (5)$$

For the subsystem of ions, in the same plasma, the relaxation time is $\tau^{(ii)} \approx \sqrt{m_i/m_e} \tau^{(ee)}$. Here, n_e, m_e, T_e are the respective electron concentration, mass and temperature and $\Lambda \approx 15$ is the “Coulomb” logarithm.

The small relaxation time and the comparatively large duration of typical thermal processes are reasons for which these important facts were not taken into account previously when constructing high-frequency heat-conduction theory.

The authors of [1, 2] predicted the existence of undamped temperature waves, which can propagate without dissipation in media with a small local temperature-relaxation time τ (the formation time of local thermodynamic equilibrium). Such waves have the characteristic eigenfrequency $\omega \geq 1/\tau$ and can be excited by the action of short thermal pulses with the duration of $\Delta t < \tau$ on the medium. The physical mechanism of the formation of these waves is related to the influence of thermal relaxation on the phase energy exchange conditions determining the energy dissipation of the thermal motion. It is obvious that the process of such an exchange is potentially reversible in a time that is less than the relaxation time.

The temperature relaxation process (without the presence of nonlocality and the time nonuniformity of the temperature establishment process) can be taken into account most simply using the modified continuity equation

$$\rho c_v \frac{\partial T(\vec{r}, t + \tau)}{\partial t} = -\text{div} \vec{q}(\vec{r}, t), \quad (6)$$

which takes into account the time delay between the local thermal energy flux and the change in the local temperature.

Combining relations (2) and (6), we find the thermal-diffusivity equation with the delay, which, in the case of the homogeneous medium, has the form

$$\frac{\partial T(x, t + \tau)}{\partial t} = G \frac{\partial^2 T(x, t)}{\partial x^2} \quad (7)$$

The solution of Eq. (7) is determined by the superposition of damped temperature waves [1, 2]:

$$\begin{aligned} T(\omega, x, t) = & A_\omega \exp\left(-\kappa \frac{\cos \omega \tau}{\sqrt{1 + \sin \omega \tau}} x\right) \\ & \times \exp\{i(\omega t - \kappa \sqrt{1 + \sin \omega \tau} x)\} \\ & + B_\omega \exp\left(\kappa \frac{\cos \omega \tau}{\sqrt{1 + \sin \omega \tau}} x\right) \\ & \times \exp\{i(\omega t + \kappa \sqrt{1 + \sin \omega \tau} x)\}, \quad \cos \omega \tau \geq 0, \end{aligned} \quad (8)$$

which are principally different from the “standard” solution (4).

For such a solution, the damping coefficient

$$\delta = \kappa \frac{\cos \omega \tau}{\sqrt{1 + \sin \omega \tau}} \equiv \sqrt{\omega/2G}(\cos \omega \tau/2 - \sin \omega \tau/2) \quad (9)$$

and the corresponding wave numbers

$$k = \kappa \sqrt{1 + \sin \omega \tau} = \sqrt{\omega/2G} \sqrt{1 + \sin \omega \tau} \quad (10)$$

turn out to be different and coincide only under the trivial condition $\tau = 0$!

This solution has physical meaning if the condition $\cos \omega \tau \geq 0$ is satisfied; as follows from (8), it corresponds to the fact that the wave can be damped only in the direction of its propagation. Thermal waves are not excited under the condition $\cos \omega \tau < 0$.

For the frequencies corresponding to the condition $\omega \tau = n\pi + \pi/2, n = 0, 1, 2, \dots$, the solution of the equations has the form of undamped forward and reverse thermal waves:

$$T(x, t) = Ae^{i(\omega t - \kappa \sqrt{2}x)} + Be^{i(\omega t + \kappa \sqrt{2}x)}, \quad (11)$$

which propagate in opposite directions with the phase velocities

$$v_n = \pm \sqrt{G\omega_n} = \pm \sqrt{G(n + 1/2)\pi/\tau}, \quad n = 0, 1, 2, \dots \quad (12)$$

It follows from these calculations that, in air, the minimum frequency of such undamped waves must correspond to $\omega_{\min} = \pi/2\tau \approx 75\text{--}85$ MHz. The results of our detailed experiments [3–6] agree well with these parameters; in this case, the distance of the reliable recording of such waves reached 2 m and was limited only by the laboratory dimensions [6].

EXCITATION METHODS AND POSSIBLE FEATURES OF THE ACTION OF AN UNDAMPED TEMPERATURE WAVE ON PHYSICAL OBJECTS

The process of undamped-wave excitation is related to possible sources of modulated thermal heating.

For example, it can be the initial source of variable local heating with a temperature variation frequency that is close to one of the optimal frequencies. Such high-frequency heating can be implemented, for example, by the action of a laser beam modulated at this frequency on the target surface.

Another variant of this heating can be a source of short thermal pulses the duration of which $\Delta t \leq \tau$ does not exceed the thermodynamic-relaxation time. Under such a condition, the required spectral components are present in the spectrum of such a pulse.

It follows from the physical essence of the process of propagation and absorption of such a wave that, at any finite distance, the initial thermal (temperature) wave is, in fact, a wave packet with the central frequency coinciding with ω_n and a narrow frequency band $\Delta\omega$ adjoining this frequency. The width of this band is determined from the condition that thermal waves with frequencies corresponding to the boundary of this interval also reach the place of recording L in the case of admissible absorption in the given medium (as a rule, it is at most doubly attenuated).

If this packet is incident at the boundary with another medium (for example, the air–metal interface), the boundary is heated very abruptly for a short time. If the packet duration is very small and its spectrum contains components with frequencies determined by the condition $\omega_n = \pi(n + 1/2)\tau, n = 0, 1, 2, \dots$, for this medium with the relaxation time τ , then analogous, but higher frequency undamped thermal waves are excited in it. Only “usual” damped thermal and acoustic waves are excited in the medium in the case of a longer packet. In addition, a shock acoustic wave which is a very narrow packet and can propagate without noticeable attenuation in this medium is generated. When this shock wave reaches, for example, the potential well, where a hydrogen or deuterium atom localized between two heavier atoms of the main lattice is located, pulse modulation of the parameters of this well occurs, namely, its very fast compression and subsequent expansion.

This process is related directly to the applied aspect of this problem, namely, features of nuclear reactions in modulated (nonstationarily deformed) potential wells. Such deformation leads to the formation of quantum coherent correlated states of light nuclei (for example, of the proton or the deuteron) located in this well. The authors of [7–11] showed that short-term particle energy fluctuations with large amplitudes are formed for such a state, which is related to the synchronization of oscillatory superposition particle

states in a nonstationary potential well. In addition to other unique features, these states are characterized by modified uncertainty relations called the Schrödinger—Robertson relations for the momentum and the coordinate

$$\delta q \delta p \geq \hbar^*/2, \quad \hbar^* = \hbar / \sqrt{1 - r_{pq}^2} \equiv G_{pq} \hbar, \quad (13)$$

$$r_{pq} = \frac{\langle qp + pq \rangle / 2 - \langle q \rangle \langle p \rangle}{\sqrt{\langle q^2 \rangle \langle p^2 \rangle}},$$

and also for the energy and the time

$$\delta E \delta t \geq \hbar^*/2, \quad \hbar^* = \hbar / \sqrt{1 - r_{Et}^2} \equiv G_{Et} \hbar, \quad (14)$$

$$r_{Et} = \frac{\langle Et + tE \rangle / 2 - \langle E \rangle \langle t \rangle}{\sqrt{\langle E^2 \rangle \langle t^2 \rangle}}.$$

In these relations, the quantity r is the correlation coefficient for the corresponding pair of dynamic variables (“energy—time” or “coordinate—momentum”) and $G = 1/\sqrt{1 - r^2}$ is the correlation-efficiency coefficient.

These relations were obtained for the first time even in 1930 [12, 13], but they were forgotten about for a number of reasons for many decades. The principal difference between these relations and the Heisenberg uncertainty relations lies in the possibility of introducing the effective Planck’s constant $\hbar^* = \hbar / \sqrt{1 - r^2} \equiv G \hbar$, the value of which can exceed the “standard” Planck’s constant \hbar by many orders.

In our papers [3–11], we showed that, in the microcrack deformation process (in fact, in the case of steady modulation of the parameters of the potential well, where deuterons are located), the quantity G can reach very large values of $G \geq 10^3 - 10^5$. It is easy to make sure that the energy-fluctuation amplitude can be very large at such values of G .

Formula (14) implies a simple estimate for the lower limit (the minimum value) of the kinetic energy fluctuation for a particle with the mass M localized within the limits of the spatial interval δq :

$$\delta E^{(\min)} \equiv (\delta p)^2 / 2M = G^2 \hbar^2 / 8M (\delta q)^2. \quad (15)$$

In particular, in the case of deuteron localization in interatomic space with a period of $a = 1.5A$, which is typical of condensed media (in this case, $\delta q \leq 0.75A$), the energy fluctuation in the coherent correlated state with $G = 10^4$ exceeds $\delta E^{(\min)} = 30$ keV. It is necessary to mention that the actual amplitude of this fluctuation can exceed this lower limit significantly, which is sufficient to implement effective nuclear fusion between different light particles (for example, deuterons) and between a specific proton or deuteron and the lattice nucleus. It is also important to note that this

fluctuation exists for a rather long time $\delta t \geq \hbar G / 2\delta E$, which increases synchronously with increasing G .

Such a general scheme can be optimized significantly if the crystal lattice containing these particles is in the stressed (quasistationary) state. In particular, in the case of a rather large degree of hydrogen and deuterium saturation of metal hydride, large internal mechanical stresses appear in it; they, reaching the critical value, can lead to the formation of microcracks and lattice cracking. The coherent correlated states of deuterons located in the volume of the formed crack also form in each event of the formation (opening) of such a microcrack. An analogous process of the formation of coherent correlated states can also be stimulated by microcrack “collapse” (“healing”). Because the number of such deuterons can be rather large in the region of the microcrack, the effect of microcrack “opening” and “collapse” can give intense ejections of particles and radiation accompanying nuclear reactions. Such a microcracking process is usually spontaneous and sporadic, and the instants of opening of different microcracks are usually independent. At the same time, a very short shock wave, which formed as a result of the thermal wave, leads to synchronization of the process of opening of such microcracks and increases sharply the efficiency of nuclear reactions.

The action of such waves can stimulate various phase transitions with a change in the local lattice topology in such a way, which can lead to the concomitant formation of coherent correlated states accompanied by the generation of giant particle energy fluctuations.

ALPHA-TRACK ANALYSIS OF REMOTE SAMPLES OF DEUTERATED TITANIUM SUBJECTED TO A THERMAL WAVE

The arguments considered above and the corresponding estimates were verified in a series of direct experiments. Samples of polycrystalline deuterated titanium with grain dimensions of at most 50 μm were chosen as the objects affected by the thermal wave. These samples had the form of cylinders with lengths of 1 cm and diameters of 7 mm. The degree of the initial deuterium saturation of these samples exceeded 150%. The geometry and the mutual arrangement of the main elements of the experimental setup are shown in Fig. 1.

The process of the cavitation of a water jet flowing through a thin channel with a diameter of about 1 mm to a cavitation chamber under pressure was used as the source of thermal waves. The initial jet pressure was 100–200 atm. In the process of the free motion of this jet, the formation and subsequent collapse of cavitation bubbles occurred; after that, the shock wave moving from it formed, first, in the liquid jet and then in the volume of the cavitation target. The reflection of this wave from the external target surface led to the

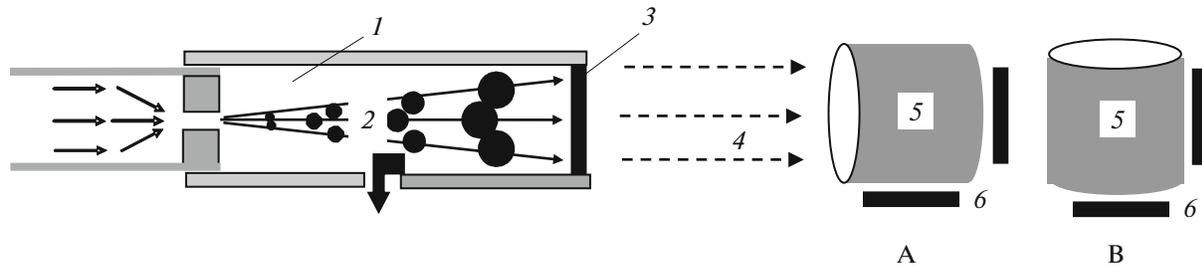


Fig. 1. Scheme of experiments for studying the influence of temperature waves on nuclear reactions in remote targets: 1, cavitation chamber; 2, water jet with cavitation bubbles; 3, cavitation target for the excitation, on the external surface, of thermal waves formed during the action of the jet with cavitation bubbles on the internal surface; 4, thermal waves propagating from the external surface of target 3; 5, remote cylindrical target (deuterated titanium); and 6, track detectors. A and B are different orientations of the same cylindrical target in different experiments.

excitation and partial ionization of surface atoms and, accordingly, to short-term pulsed heating of the surface air layer. The generation of soft X-ray radiation (in the range of 1–5 keV depending on the type of surface atom) is confirmation of such a scenario [14, 15].

The pulse duration of such heating was determined by the very small extent of the shock-wave front. It is very important to note that the process of cavitation-bubble formation was ordered rather than random. This was due to positive feedback, which formed because of the action of shock waves on the parameters of the initial channel, through which the water flows into the chamber under pressure. Such a system is analogous to the known Van der Pol generator formed because of positive feedback in vacuum tubes and leading to electron-flux modulation. In the given case, the cavitation bubbles were direct analogues of electrons.

To perform alpha-track analysis, we used a plastic detector of the CR-39 type with a density of 1.3 g/cm³. The TASTRAK® (Track Analysis Systems Ltd, Bristol, Great Britain) detector sheet thickness was 1 mm.

The typical experimental arrangement corresponded to a detector location at a distance of 5 mm from the target surface, onto which the thermal wave was incident for a certain time (for example, 20 and 40 min). The target was located at a distance of about 20 cm from the external surface of the cavitation chamber. The studies were conducted for two variants of cylindrical-target orientations (A and B) with respect to the direction of the thermal-wave motion. Both variants are presented in Fig. 1.

After irradiation, the detector was subjected to 6.25-M etching by a NaOH solution for 3.5 h at a temperature of 80°C. Alpha-track images were photographed using an OlympusBX-51 optical microscope with an ImageScopeM program. The particle-motion trajectories were determined using the alpha-track positions and directions.

Figures 2a and 2b show photographs of large fragments of a solid-state track detector located near the end and side sample surfaces and placed at position A at a distance of about 20 cm from the external surface of the cavitation chamber during one of a series of actions of the thermal wave on it.

The data given in Fig. 2a correspond to the case of the geometry where the detector was placed at a distance of 5 mm behind the rear (with respect to the thermal-wave source) end sample surface. Figure 2b shows a photograph of a fragment of the track detector, which was located parallel to the side surface of the cylindrical sample at an analogous distance.

Both detectors were exposed for the same time (40 min).

For the control, Fig. 2c shows a fragment of an analogous track detector irradiated with the reference source of “triplet” (U²³³, Pu²³⁸, and Pu²³⁹) alpha radiation.

In Fig. 2a, the motion direction for the main groups of particles is denoted by arrows. It follows from the form of the track detector corresponding to the symmetric arrangement of the detector with respect to the end sample surface that the motion trajectory of recorded particles is also characterized by central symmetry, which agrees well with the assumption of the axially symmetric separation of nuclear-reaction products. It is important to note that the parameters and the main characteristics of the alpha tracks were almost identical in the experimental and reference detectors.

Figure 3 shows the results of an analogous study of alpha-particle fluxes formed in the case of arrangement of the target in the geometry B (Fig. 1) and of track detectors near the flat and cylindrical target surfaces at a distance of 5 mm, respectively. It follows from these data that the effectiveness of the influence of the thermal wave is significantly smaller for such a target orientation.

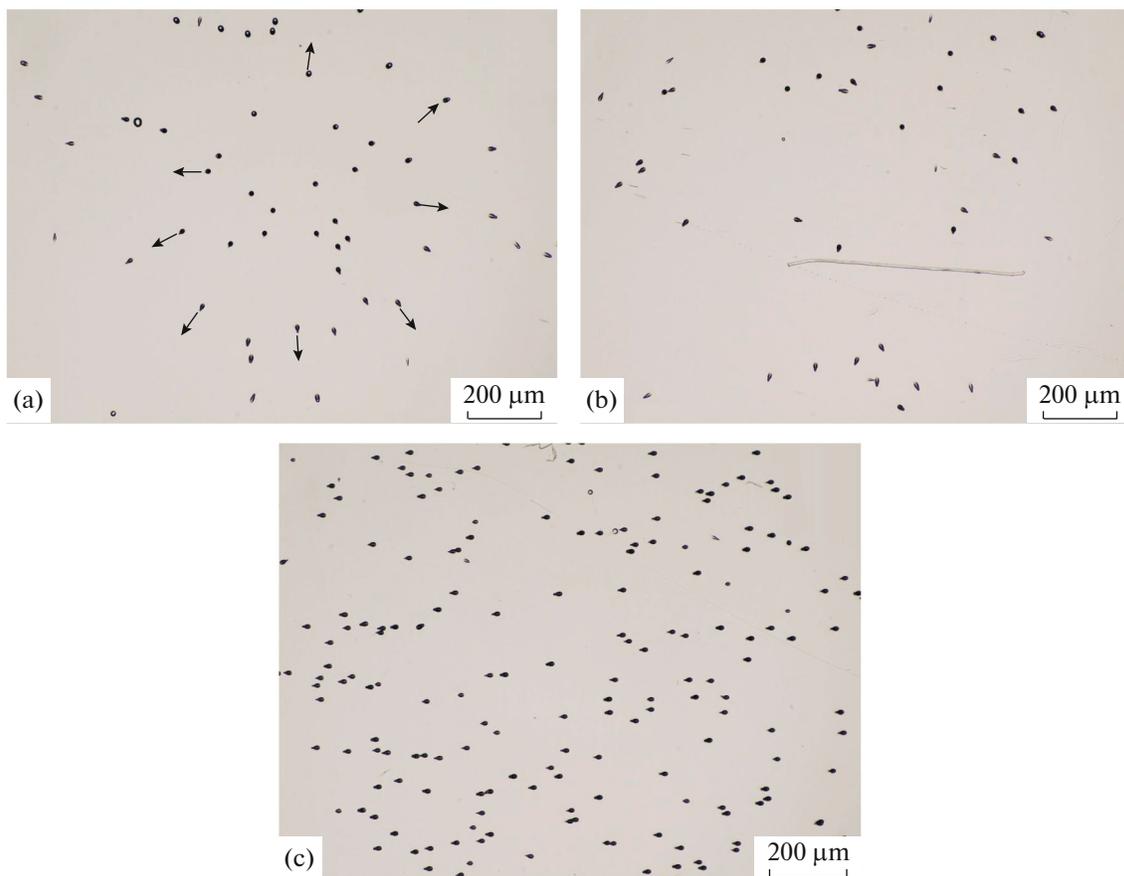


Fig. 2. Track analysis of the spatial distribution and the direction of alpha-particle motion in samples subjected to a thermal wave for 40 min in the case of the location of the track detector near the end (Fig. 2a) and side (cylindrical) (Fig. 2b) surfaces of the deuterated sample oriented at position A (Fig. 1). Fig. 2c corresponds to the reference irradiation of an analogous track detector with the use of an alpha radiator on the basis of the combination of three radionuclides U^{233} , Pu^{238} , and Pu^{239} .

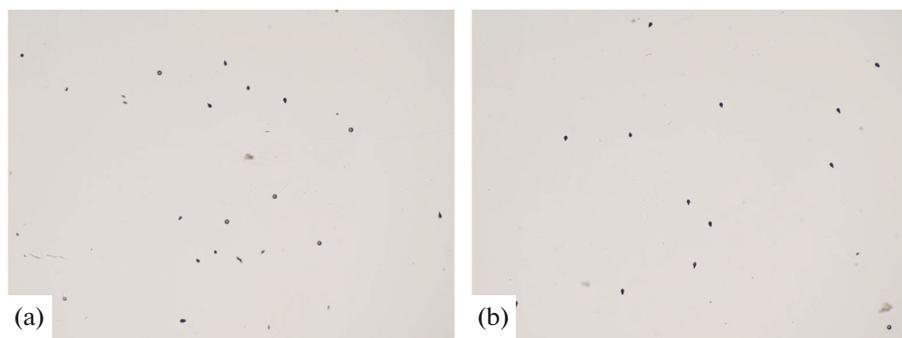


Fig. 3. Tracks at the track detectors formed for target orientation in position B (Fig. 1) and under irradiation with a thermal wave. The upper image (a) corresponds to the position of the track detector near the end (flat) target surface; and the lower image (b), near the cylindrical surface.

CONCLUSIONS

The problems of the type of nuclear reaction observed in these experiments and also of the types of particles recorded using track detectors are very important for understanding the mechanism of the

observed process. It is known that, in the case of a large deuteron energy, the two following reactions have great (and approximately the same) probabilities:



$$d + d = n + \text{He}^3 + 3.27 \text{ MeV.} \quad (17)$$

The cross sections for these reactions are 0.09 b.

The third possible reaction

$$d + d = \text{He}^4 + 23.8 \text{ MeV,} \quad (18)$$

has a very small probability for a large energy of interacting particles (its cross section is 10^{-26} b). The case can change at a low energy. In this case, the interaction process is determined by the virtual energy (giant energy fluctuations) formed as a result of the formation of coherent correlated states and released in a reversible way in a comparatively short time rather than the actual energy of interacting particles (it is small for such reactions between charged particles). In this case, the lifetime of the formed composite nucleus for a specific nuclear-fusion channel is very important (the shorter it is, the more effective the process). These problems were considered in more detail in [8–11] and will be studied later with regard to the given experiments.

The method for comparing the forms of tracks using experimental and reference detectors is rather effective at the given stage of studies. The very high degree of track identity is evidence of the fact that alpha particles of the He^3 and He^4 types and, probably, fast protons and tritons were also recorded in the given experiment. Some of the features of these experiments were also discussed in [16].

The given studies will be continued; however, it is obvious even at this stage that the method of such remote stimulation of nuclear fusion offers new opportunities and prospects for the implementation of controlled nuclear fusion [17–20].

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