Effect of impurities on nuclear fusion

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Modification of nuclear reactions due to impurities in plasma is investigated. The hindering effect of Coulomb repulsion between reacting particles, that is effective in direct reactions, can practically disappear if Coulomb interaction of either of the reacting particles with impurities embedded in plasma is taken into account. The description (based on standard second order time independent perturbation calculation of quantum mechanics) can be interpreted as if a slow, quasi-free particle (e.g. a proton) were pushed by a heavy, assisting particle (impurity) of the surroundings and can get (virtually) such a great magnitude of momentum which significantly increases the probability of its capture by an other nucleus. As a sample reaction the process, called impurity assisted nuclear *pd* reaction is investigated and the rate and power produced by the reaction are numerically calculated. A partial survey of impurity assisted nuclear reactions which may have practical importance in energy production is also presented.

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I. INTRODUCTION

It is a commonplace that because of the Coulomb repulsion between the nuclei nuclear fusion reactors need to be heated to very high temperature to be ignited [1]. (Mathematically it is manifested in the exponential energy dependent factor in the cross section of fusion reactions [see (23)]. For details see the Appendix.)

In astrophysical condensed and dense laboratory plasmas the effect of the surroundings on the nuclear fusion rate is important. In tenuous plasmas the effect of spectator nuclei and electrons (the environment) on the Gamow-rate of reacting nuclei which are assumed to interact with bare Coulomb potential is negligible [2]. Moreover in (e.g. tokamak-like) devices the presence of impurities during the heating up and working periods is undesirable because of high loss power generated by them [3].

In this paper it will be shown, however, that spectator nuclei may considerably affect nuclear reactions that allows new types of reactions and what is more, the mechanism found does not need plasma state at all.

We are going to investigate processes that can take place due to impurities and their effect on fusion reactions. We focus our attention on the Coulomb interaction between the fuel nuclei and the environment, namely on consequences of interactions with impurities that can change the fusion rate.

We investigate the

$$ {}_{z_1}^{A_1}V + p + {}_{z_3}^{A_3}X \rightarrow {}_{z_1}^{A_1}V' + {}_{z_3+1}^{A_3+1}Y + \Delta \qquad (1) $$

process called impurity assisted proton capture, a process among atoms or atomic ions containing ${}_{z_1}^{A_1}V$ nuclei (e.g. Xe) and protons or hydrogen atoms and ions or atoms of nuclei ${}_{z_3}^{A_3}X$ (e.g. deuterons) and initially it is supposed that the material is in plasma state. Δ is the energy of the reaction. First we pay our attention to the impurity assisted $p + d \rightarrow {}^3He$ reaction

$$ {}_{z_1}^{A_1}V + p + d \rightarrow {}_{z_1}^{A_1}V' + {}_{2}^{3}He + \Delta \qquad (2) $$

in an impurity contaminated plasma, that will be discussed in more detail. Here $\Delta = 5.493\ MeV$. It is worth mentioning that in the usual

$$ p + d \rightarrow {}_{2}^{3}He + \gamma + 5.493 MeV \qquad (3) $$

reaction particles ${}_{2}^{3}He$ and γ take away the reaction energy and electromagnetic interaction governs the reaction. In the impurity assisted $p + d \rightarrow {}^3He$ reaction (in reaction (2)) particles ${}_{z_1}^{A_1}V'$ and ${}_{2}^{3}He$ carry away the reaction energy while Coulomb and strong interactions govern the reaction. It is worth mentioning too that calculations indicate that in process (1) the cross section of the indirect (second order) reaction may be essentially higher with decreasing energy than the cross section of the direct (first order) reaction and the huge exponential drop in the cross section (23) with decreasing energy may disappear. Consequently, the plasma temperature can be significantly reduced.

The rate of the process to be considered can be calculated according to the rules of second order time-independent perturbation calculation of quantum mechanics [4]. The corresponding graphs can be seen in Fig. 1. which can help to understand the explanation of the effect. The physics behind the calculation may be interpreted in the following manner. The Coulomb interaction between particles ${}_{z_1}^{A_1}V$ and protons mixes (an intermediate) state of the proton of large momentum to the initially slow protonic state with a small amplitude while the particle ${}_{z_1}^{A_1}V$ is recoiled. Thus the Coulomb

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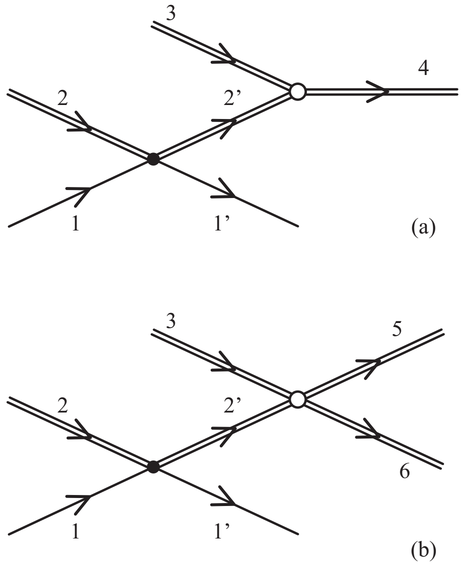

FIG. 1: The graphs of impurity assisted nuclear reactions. The single lines represent (initial (1) and final (1')) impurity particle of the plasma. The double lines represent free, heavy initial (2) particles (such as p, d), their intermediate state (2'), target nuclei (3) and reaction products (4, 5, 6). The filled dot denotes Coulomb-interaction and the open circle denotes nuclear (strong) interaction. FIG. 1(a) is a capture process and FIG. 1(b) is a reaction with two fragments.

interaction pushes the protons (virtually) into an intermediate state. In this state protons have large enough (virtual) momentum to get over the Coulomb repulsion of nuclei ${}^{A_3}_{z_3}X$ and so they may captured by the nuclei ${}^{A_3}_{z_3}X$ due to strong interaction to create a ${}^{A_3+1}_{z_3+1}Y$ nucleus. The particles (impurities) ${}^{A_1}_{z_1}V$ (initial) and ${}^{A_1}_{z_1}V'$ (final) assist the process only. (Details of calculations can be found in the Appendix.)

The virtual momentum of the intermediate state can be determined in the following way. Energy and momentum conservations determine the wave-vectors $\mathbf{k}_{1'}$ and $\mathbf{k}_4$, which are wave vectors of particles 1' and 4, respectively, as $\mathbf{k}_{1'} = -\mathbf{k}_4$ and $|\mathbf{k}_{1'}| = |\mathbf{k}_4| = k_0$ with $k_0 = \hbar^{-1}\sqrt{2m_0 a_{14}\Delta}$. Here $m_0c^2 = 931.494\ MeV$ is the atomic energy unit and a_{14} is determined by (25). (It is assumed that initial momenta and kinetic energies are negligible.) Because of momentum conservation in Coulomb scattering of plane waves the wave vector $\mathbf{k}_{2'}$ of particle 2' is determined as $\mathbf{k}_{2'} = -\mathbf{k}_{1'}$, i.e. $|\mathbf{k}_{2'}| = k_0$ too. Consequently $|\mathbf{k}_{2'}|$ is large enough for particle 2' to effectively overcome the Coulomb repulsion.

A generalization of (1) is the reaction

$$ {}^{A_1}_{z_1}V + {}^{A_2}_{z_2}w + {}^{A_3}_{z_3}X \to {}^{A_1}_{z_1}V' + {}^{A_3+A_2}_{z_3+z_2}Y + \Delta \tag{4} $$

that will be briefly discussed to draw conclusions as to the possible modification of appropriate fuels of nuclear fusion reactors by impurity assisted reactions.

The Section II. is devoted to the discussion of rate and power of impurity assisted $p+d \to {}^3He$ reaction, which is the simplest impurity assisted proton capture reaction, in an atomic-atom ionic gas mix. (The physical background namely the so-called Coulomb factor, general considerations about impurity assistance, applied model of impurity assisted nuclear p-capture reactions and basis of rate and power calculations can be found in the Appendix (Section VI.). Section III. is a partial overview of some other impurity assisted low energy nuclear reactions. In Section IV. the Xe-atomic Li mixture is discussed which may be useful in nuclear energy production in the future. Section V. is a Summary.

II. RATE AND POWER IN A $p-d-Xe$ ATOMIC ATOM-IONIC GAS MIX

Reaction (3) is not suitable for energy production since its cross section (the $S(0)$ value, see [5]) is rather small compared to other candidate reactions and only a minor part of the reaction energy $\Delta = 5.493\ MeV = 8.800 \times 10^{-13}\ J$ is taken away by ${}^3He$ ($E_{{}^3He} = \Delta^2/\left(6m_0c^2\right) = 5.4\ keV$) and the main part $E_\gamma = 5.488\ MeV$ is taken away by γ radiation which is difficult to convert to heat. But in the impurity assisted version of the $p + d \to {}^3_2He$ reaction in the

$$ {}^{A_1}_{z_1}V + p + d \to {}^3_2He + {}^{A_1}_{z_1}V' + 5.493MeV \tag{5} $$

reaction the reaction energy is taken away by particles ${}^3_2He$ and ${}^{A_1}_{z_1}V'$ as their kinetic energy that they can lose in a very short range to their environment converting the reaction energy efficiently into heat. This reaction can happen in an atomic-atom ionic gas mix (briefly called special gas environment in the following). Moreover, as was said earlier the reaction (3) is governed by electromagnetic interaction and reaction (5) happens due to Coulomb and strong interactions.

If deuterons are present in the special gas environment then the following

$$ {}^{A_1}_{z_1}V + d + d \to {}^{A_1}_{z_1}V' + {}^4_2He + 23.847MeV, \tag{6} $$

$$ {}^{A_1}_{z_1}V + d + d \to {}^{A_1}_{z_1}V' + n + {}^3_2He + 3.269\ MeV \tag{7} $$

and

$$ {}^{A_1}_{z_1}V + d + d \to {}^{A_1}_{z_1}V' + p + t + 4.033\ MeV \tag{8} $$

impurity assisted dd reactions may also take place. In these reactions the energy of the reaction is carried by particles ${}^{A_1}_{z_1}V'$ and ${}^4_2He$ which have momentum of equal magnitude but opposite direction in the case of (6), by particles ${}^{A_1}_{z_1}V'$, n and ${}^3_2He$ in the case of (7) and by particles ${}^{A_1}_{z_1}V'$, p and t in the case of (8).

The rate (dn_f/dt) and power ($\Delta dn_f/dt$) densities of impurity assisted $p + d \to {}^3_2He$ reaction are determined by (43) and (44) (see the Appendix) with

$$ S = 1.96 \times 10^{-54} z_1^2\ cm^6 s^{-1}, \tag{9} $$

where z_1 is the charge number of the assisting nucleus and with

$$P = 1.72 \times 10^{-66} z_1^2 \ cm^6 W, \quad (10)$$

respectively. Taking $z_1 = 54$ (Xe) and $n_1 = n_2 = n_3 = 2.65 \times 10^{20}$ cm^{-3} (n_1, n_2 and n_3 are the number densities of Xe, p and d, i.e. particles 1, 2 and 3) one gets

$$dn_f/dt = 1.06 \times 10^{11} \ cm^{-3} s^{-1} \quad (11)$$

and

$$\Delta dn_f/dt = 0.093 \ W cm^{-3} \quad (12)$$

rate and power densities of considerable value. If the impurity is Hg or U then the above numbers must be multiplied by 2.2 or 2.9, respectively.

One must emphasize that both rate and power densities (dn_f/dt and p) are temperature independent. It must be mention too that the effect does not affected by the Coulomb screening and the only condition is that the participants must be in atomic or in atom-ionic state. This condition and the temperature independence of dn_f/dt and p greatly weaken necessary experimental conditions.

III. OTHER IMPURITY ASSISTED NUCLEAR REACTIONS

Now let us consider the impurity assisted proton captures [see (1) and Fig. 1a] in general. The reaction energy Δ is the difference between the sum of the initial and final mass excesses, i.e. $\Delta = \Delta_p + \Delta_{A_3,z_3} - \Delta_{A_3+1,z_3+1}$. Since particle 1 assists the nuclear reaction its rest mass does not change. Δ_p, Δ_{A_3,z_3} and Δ_{A_3+1,z_3+1} are mass excesses of proton, ${}^{A_3}_{z_3}X$ and ${}^{A_3+1}_{z_3+1}Y$ nuclei, respectively [6]. Moreover, the capture reaction may be extended to the impurity assisted capture of particles ${}^{A_2}_{z_2}w$ (see reaction (4)), e.g. the capture of deuteron (d), triton (t), ${}^3He$, ${}^4He$, etc.. In this case $\Delta = \Delta_{A_2,z_2} + \Delta_{A_3,z_3} - \Delta_{A_3+A_2,z_3+z_2}$. Δ_{A_2,z_2}, Δ_{A_3,z_3} and $\Delta_{A_3+A_2,z_3+z_2}$ are the corresponding mass excesses. Investigating the mass excess data [6] one can recognize that in the case of both processes the number of energetically allowed reactions is large, their usefulness from the point of view of energy production is mainly determined by the magnitude of the numerical value of the factor $f^2_{2'3}$ (see the Appendix) which belongs to the actual reaction.

Impurity $\left({}^{A_1}_{z_1}V\right)$ assisted $d-Li$ reaction may take place with ${}^6_3Li$ and ${}^7_3Li$ isotopes:

$${}^{A_1}_{z_1}V + d + {}^6_3Li \rightarrow {}^{A_1}_{z_1}V' + 2{}^4_2He + 22.372 \ MeV, \quad (13)$$

$${}^{A_1}_{z_1}V + d + {}^7_3Li \rightarrow {}^{A_1}_{z_1}V' + 2{}^4_2He + n + 15.122 \ MeV \quad (14)$$

and

$${}^{A_1}_{z_1}V + d + {}^7_3Li \rightarrow {}^{A_1}_{z_1}V' + {}^9_4Be + 16.696 \ MeV. \quad (15)$$

If there are deuterons present then the

$${}^{A_1}_{z_1}V + d + {}^{A_3}_{z_3}X \rightarrow {}^{A_1}_{z_1}V' + {}^{A_3+2}_{z_3+1}Y + \Delta \quad (16)$$

impurity assisted d capture process (see e.g. (15)) and if there is Li present then

$${}^{A_1}_{z_1}V + {}^{A_2}_{3}Li + {}^{A_3}_{z_3}X \rightarrow {}^{A_1}_{z_1}V' + {}^{A_2+A_3}_{z_3+3}Y + \Delta \quad (17)$$

impurity assisted Li capture reactions may happen.

IV. Xe-ATOMIC Li MIXTURE - A CANDIDATE FOR NUCLEAR FUEL

Let us examine the impurity assisted

$${}^{A_1}_{z_1}V + {}^{A_2}_{3}Li + {}^{A_3}_{3}Li \rightarrow {}^{A_1}_{z_1}V' + {}^{A_2+A_3}_{6}C + \Delta \quad (18)$$

Li capture reactions. Using (29) with $z_2 = z_3 = 3$, $A_2 = 6$, $A_3 = 7$, $A_2 + A_3 = A_4 = 13$ which corresponds to the

$${}^{A_1}_{z_1}V + {}^6_3Li + {}^7_3Li \rightarrow {}^{A_1}_{z_1}V' + {}^{13}_{6}C + 25.869 \ MeV \quad (19)$$

reaction, and taking $A_1 = 130$, $\eta_{2'3} = 0.486$ and $f^2_{2'3} = 0.151$.

The reactions

$${}^{A_1}_{z_1}V + {}^6_3Li + {}^6_3Li \rightarrow {}^{A_1}_{z_1}V' + {}^{12}_{6}C + 28.174 \ MeV, \quad (20)$$

$${}^{A_1}_{z_1}V + {}^6_3Li + {}^6_3Li \rightarrow {}^{A_1}_{z_1}V' + 3{}^4_2He + 20.898 \ MeV, \quad (21)$$

$${}^{A_1}_{z_1}V + {}^7_3Li + {}^7_3Li \rightarrow {}^{A_1}_{z_1}V' + {}^{14}_{6}C + 26.795 \ MeV \quad (22)$$

may have importance too. (This list of reactions is incomplete. Reactions of smaller reaction energy are omitted.)

Since the screening of the Coulomb potential is not essential ($k_0 \gg q_{sc}$, see the Appendix) the above reactions bring up the possibility of a quite new type of apparatus since the processes need atomic state of participant materials only. Thus e.g. a $Xe - atomic \ Li$ mixture may be appropriate which needs very low temperature compared to the working temperature of fusion power stations planned these days as the boiling temperature of metallic Li is $T = 1615 \ K$ at normal pressure.

V. SUMMARY

The consequences of impurities in nuclear fusion fuels of plasma state are discussed. According to calculations in certain cases second order processes may produce greatly higher fusion rate than the rate due to direct (first order) processes. In the examined problem it is found that Coulomb scattering of the fusionable nuclei on the screened Coulomb potential of the impurity can diminish the hindering Coulomb factor between them. Since the second order process does not demand the matter to be in ionized state the assistance of impurities can allow to decrease significantly the plasma temperature which is determined only by the requirement that all components must be in atomic or atom-ionic state. The results suggest that, on the other hand, the density of the components has to be considerably increased. Promising new fuel mixes are also put forward. On the base of these results it may be expected that new implementation for energy production by nuclear fusion may be found.

VI. APPENDIX

A. Coulomb factor and general considerations

The cross section (σ) of usual fusion reactions reads as [5]

$$\sigma\left(E\right)=S\left(0\right)\exp\left[-2\pi\eta_{jk}\left(E\right)\right]/E, \tag{23}$$

where

$$\eta_{jk}\left(E\right)=z_{j}z_{k}\alpha_{f}\sqrt{a_{jk}\frac{m_{0}c^{2}}{2E}} \tag{24}$$

is the Sommerfeld parameter. z_j and z_k are the charge numbers,

$$a_{jk}=\frac{A_{j}A_{k}}{A_{j}+A_{k}} \tag{25}$$

is the reduced mass number of mass numbers A_j and A_k, and E is the energy of the relative motion of the reacting particles of rest masses $m_j = A_j m_0$, $m_k = A_k m_0$. $m_0 c^2 = 931.494$ MeV is the atomic energy unit, α_f is the fine structure constant. E is taken in the center of mass (CM) coordinate system. $S(0)$ is the astrophysical factor at $E = 0$.

The cross section (23) can be derived applying the Coulomb solution $\varphi_{Cb}(\mathbf{r})$, which is the wave function of a free particle of charge number z_j in a repulsive Coulomb field of charge number z_k [7], in the description of relative motion of projectile and target. $\varphi_{Cb}(\mathbf{r}) \sim e^{-\pi\eta_{jk}/2}\Gamma(1+i\eta_{jk})$ and

$$\left|e^{-\pi\eta_{jk}/2}\Gamma(1+i\eta_{jk})\right|=\sqrt{\frac{2\pi\eta_{jk}\left(E\right)}{\exp\left[2\pi\eta_{jk}\left(E\right)\right]-1}}=f_{jk}(E), \tag{26}$$

which is the Coulomb factor. (23) is proportional to $f_{jk}^2(E)$ and one can show that the exponential factor in (23) comes from $f_{jk}^2(E)$. Thus the smallness of rate at low energies is the consequence of $f_{jk}^2(E)$ becoming very small at these energies. So the magnitude of the Coulomb factor $f_{jk}(E)$ is crucial from the point of view of magnitude of the cross section and therefore we concentrate on it.

Taking into account the Coulomb repulsion between particles $2'$ and 3 an approximate form

$$\varphi(\mathbf{r})=f_{jk}(k\left[E\right])\exp(i\mathbf{k}\cdot\mathbf{r})/\sqrt{V} \tag{27}$$

of the Coulomb solution $\varphi_{Cb}(\mathbf{r})$ valid in the nuclear volume is used that produces the same $\left|\varphi(\mathbf{0})\right|^2 = f_{jk}^2/\sqrt{V}$ contact probability density as $\varphi_{Cb}(\mathbf{r})$.

Let us consider the impurity assisted $p + d \rightarrow {}_2^3He$ model-reaction in a plasma with Xe admixture. (It is an example of the general impurity assisted $p + {}_{z_3}^{A_3}X \rightarrow {}_{z_4}^{A_4}Y$ process with $A_4 = A_3 + 1$, $z_4 = z_3 + 1$). The particles (Xe, proton, d and ${}_2^3He$) must fulfill energy and momentum conservation in their initial and final states. As a consequence, the energy $E_{2'}(CM)$ of the pushed proton in the CM system of particles $2'$ and 3 where the Coulomb factor must be calculated is

$$E_{2'}(CM)=\frac{A_{3}}{A_{2}\left(A_{2}+A_{3}\right)}a_{14}\Delta. \tag{28}$$

Substituting $E_{2'}(CM)$ in (24)

$$\eta_{2'3}=z_{2}z_{3}\alpha_{f}A_{2}\sqrt{\frac{m_{0}c^{2}}{2a_{14}\Delta}}. \tag{29}$$

It gives $\eta_{2'3} = z_3\alpha_f\sqrt{m_0c^2/\left(2a_{14}\Delta\right)}$ with $z_2 = 1$, $A_2 = 1$. A_1 is the nucleon number of Xe and $z_3 = 1$, $A_4 = 3$ corresponding to impurity assisted $p + d \rightarrow {}_2^3He$ reaction with particle 2 is the proton. In this reaction $\Delta = 5.493$ MeV and if $A_1 = 130$ which is a typical mass number of the Xe isotopes then $2a_{14}\Delta = 32.21$ MeV, $\eta_{2'3} = 0.039$ and $f_{2'3}^2 = 0.88$. If particle 2 is a deuteron ($z_2 = 1$) then $A_2 = 2$ resulting $\eta_{2'3} = 2z_3\alpha_f\sqrt{m_0c^2/\left(2a_{14}\Delta\right)} = 0.078$ and $f_{2'3}^2 = 0.77$.

Thus, the Coulomb scattered proton will have (virtual) momentum (virtual kinetic energy) in the intermediate state that is large enough to make a drastic increase of the Coulomb factor $f_{2'3}$ as has been demonstrated.

B. Model of impurity assisted nuclear p-capture reactions in an atomic-atom ionic gas mix

The reaction of heavy particles in the special gas environment is modeled in the following way. The unperturbed Hamiltonian contains the kinetic part only, a plausible choice because of the screening due to the plasma and the remaining electrons of ions. Accordingly, the initial, intermediate and final states of all components of the special gas environment, which is supposed to be electrically neutral, are all plane waves. The interaction Hamiltonian H_I which describes the interaction between the free particles has the form

$$H_{I}=V_{Cb}+V_{St}. \tag{30}$$

where V_{Cb} is the screened Coulomb interaction potential and V_{St} is the interaction potential of the strong interaction. Therefore according to (30), the lowest order of

S-matrix element of impurity assisted nuclear reaction, which is at least of second order in terms of standard perturbation calculation, has two terms (see Fig. 1a). In the following we only deal with the dominant term which results the transition rate [4] as

$$W_{fi}^{(2)}=\frac{2\pi}{\hbar}\sum_{f}\left|T_{fi}^{(2)}\right|^{2}\delta(E_{f}-\Delta) \tag{31}$$

with

$$T_{fi}^{(2)}=\sum_{2'}\frac{V_{St,f2'}V_{Cb,2'i}}{E_{2'}-E_{i}}. \tag{32}$$

Here Δ is the reaction energy, i.e. the difference between the rest energies of the initial (E_{i0}) and final (E_{f0}) states ($\Delta=E_{i0}-E_{f0}$). E_i, $E_{2'}$ and E_f are the kinetic energies in the initial, intermediate and final states, respectively. The initial momenta and kinetic energies of particles 1, 2 and 3 are neglected ($E_i=0$).

$$E_{2'}=\frac{\hbar^{2}\mathbf{k}_{2'}^{2}}{2m_{2}}+\frac{\hbar^{2}\mathbf{k}_{1'}^{2}}{2m_{1}},\ E_{f}=\frac{\hbar^{2}\mathbf{k}_{1'}^{2}}{2m_{1}}+\frac{\hbar^{2}\mathbf{k}_{4}^{2}}{2m_{4}} \tag{33}$$

and $\mathbf{k}_{1'}$, $\mathbf{k}_{2'}$ and $\mathbf{k}_4$ are wave vectors of particle 1', 2' and 4, respectively. It follows from the calculation that finally $|\mathbf{k}_{1'}|=|\mathbf{k}_{2'}|=|\mathbf{k}_4|=k_0$, and k_0 see below. Consequently the screening (see below) will not have essential role since $k_0\gg q_{sc}$. Only capture processes (see Fig. 1a) are dealt with in detail.

The screened Coulomb interaction potential $V_{Cb}(\mathbf{r})$ with charge number z_1 and screening parameter $q_{sc}=z_1/a_0\gg q_D$ (q_D the plasma Debye screening parameter and $z_1>z_2$, z_3) has the form

$$V_{Cb}(\mathbf{r})=\frac{z_{1}z_{2}}{2\pi^{2}}e^{2}\int\frac{\exp(i\mathbf{qr})}{q^{2}+q_{sc}^{2}}d\mathbf{q}. \tag{34}$$

Here the coupling strength $e^2=\alpha_f\hbar c$. $\hbar$ is the reduced Planck-constant, c is the velocity of light in vacuum, a_0 is the Bohr radius and z_2 is the charge number of the other heavy particle (particle 2).

In the nuclear part of the model the interaction potential $V_{St}(\mathbf{x})=-V_0$ if $|\mathbf{x}|\leq b$ and $V_{St}(\mathbf{x})=0$ if $|\mathbf{x}|>b$, with $V_0=25\ MeV$ and $b=2\times10^{-13}\ cm$ in the case of deuteron target [8], and the Weisskopf-approximation are applied, i.e. for the final nuclear state of the proton we take $\Phi_{fW}(\mathbf{r})=\sqrt{3/(4\pi R_0^3)}$ if $r\leq R_0$, where R_0 is the nuclear radius, and $\Phi_{fW}(\mathbf{r})=0$ for $r>R_0$ and assume that $R_0=b$. The Coulomb repulsion between particles $2'$ and 3 is taken into account using (27).

The Coulomb matrix element with plane waves is

$$V_{Cb,2'i}=\frac{z_{1}z_{2}}{2\pi^{2}}e^{2}\frac{\delta\left(\mathbf{k}_{2'}+\mathbf{k}_{1'}\right)}{\mathbf{k}_{1'}^{2}+q_{sc}^{2}}\frac{\left(2\pi\right)^{6}}{V^{2}} \tag{35}$$

where V is the volume of normalization.

The matrix element of the strong interaction potential between particles 2' and 3 is

$$\begin{aligned}V_{St,f2'}^{W} &= -V_{0}\frac{\sqrt{12\pi R_{0}}}{k_{2'}}f_{2'3}(k_{2'})\times \\ &\times H\left(k_{2'}\right)\frac{3}{A_{2}}\frac{\left(2\pi\right)^{3}}{V^{3/2}}\delta\left(\mathbf{k}_{2'}-\mathbf{k}_{4}\right),\end{aligned} \tag{36}$$

where

$$H\left(k_{2'}\right)=\int_{0}^{1}\sin(k_{2'}R_{0}\frac{A_{2}}{3}x)xdx \tag{37}$$

and (27) is used.

In the case of impurity assisted $p+d\rightarrow{}_{2}^{3}He$ reaction when calculating the transition rate one has two terms from two initial states the one of which comes from $z_2=1$, $A_2=1$ (particle 2 is a proton and particle 3 is deuteron) and the other one comes from $z_2=1$, $A_2=2$ (particle 2 is a deuteron and particle 3 is proton). Consequently the transition probability per unit time of the impurity assisted pd reaction in a special gas environment reads as

$$W_{fi}^{(2)}=\frac{216\pi^{2}\sqrt{2}}{a_{14}^{7/2}V^{2}}cR_{0}\frac{z_{1}^{2}\alpha_{f}^{2}V_{0}^{2}\left(\hbar c\right)^{4}}{\Delta^{9/2}\left(m_{0}c^{2}\right)^{3/2}}F_{2'3} \tag{38}$$

with $F_{2'3}=\frac{1}{2}\sum_{A_2=1,2}a_{12}^2f_{2'3}^2(k_0)H^2(k_0)/A_2^2$ and $k_0=\hbar^{-1}\sqrt{2m_0a_{14}\Delta}$.

C. Rate and power densities

The cross section $\sigma^{(2)}$ of the process can be determined from $W_{fi}^{(2)}$ in the usual manner as

$$\sigma^{(2)}=\frac{N_{1}W_{fi}^{(2)}}{\frac{v_{23}}{V}}, \tag{39}$$

where N_1 is the number of particles in the normalization volume V and v_{23} is the relative velocity of particles 2 and 3. Thus

$$v_{23}\sigma^{(2)}=n_{1}S \tag{40}$$

with n_1 the number density of impurity, i.e. particles 1 ($n_1=N_1/V$) and

$$S=\frac{216\pi^{2}\sqrt{2}}{a_{14}^{7/2}}cR_{0}\frac{z_{1}^{2}\alpha_{f}^{2}V_{0}^{2}\left(\hbar c\right)^{4}}{\Delta^{9/2}\left(m_{0}c^{2}\right)^{3/2}}F_{2'3}, \tag{41}$$

which is temperature independent. The rate dN_f/dt in the whole volume V can be written as

$$\frac{dN_{f}}{dt}=N_{3}\Phi_{23}\sigma^{(2)} \tag{42}$$

where $\Phi_{23}=n_2v_{23}$ is the flux of particles 2 with n_2 their number density ($n_2=N_2/V$) and N_3 is the number of particles 3 in the normalization volume. The rate density $dn_f/dt = V^{-1}dN_f/dt$ of the process can be written as

$$\frac{dn_f}{dt} = n_3 n_2 n_1 S, \tag{43}$$

where n_3 is the number density of particles 3 ($n_3 = N_3/V$). The power density is defined as

$$p = \Delta \frac{dn_f}{dt} = n_1 n_2 n_3 P \tag{44}$$

with $P = S\Delta$. The rate and power densities (dn_f/dt and p) are temperature independent.

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