Electron assisted neutron exchange process in solid state environment

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Abstract

Electron assisted neutron exchange process in solid state environment is investigated. It is shown that if a metal is irradiated with free electrons then the $e+ {}^{A_1}_{Z}X+ {}^{A_2}_{Z}X \rightarrow e'+ {}^{A_1-1}_{Z}X+ {}^{A_2+1}_{Z}X+\Delta$ electron assisted neutron exchange process has measurable probability even in the case of slow electrons of energy much less than the reaction energy Δ. The transition probability per unit time, the cross section of the process and the yield in an irradiated sample are determined in the Weisskopf and long wavelength approximations and in the single particle shell model. Numerical data for the $e+ {}^{A_1}_{28}Ni+ {}^{A_2}_{28}Ni \rightarrow e'+ {}^{A_1+1}_{28}Ni+ {}^{A_2-1}_{28}Ni+\Delta$ and the $e+ {}^{A_1}_{46}Pd+ {}^{A_2}_{46}Pd \rightarrow e'+ {}^{A_1+1}_{46}Pd+ {}^{A_2-1}_{46}Pd+\Delta$ electron assisted neutron exchange reactions are also presented.

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I. INTRODUCTION

In the last two decades extraordinary observations were made in low energy accelerator physics. Namely the so called anomalous screening effect was observed investigating astrophysical factors of nuclear reactions of low atomic numbers mostly in deuterated metal targets [1].

Motivated by these observations we have searched physical processes that may effect nuclear reactions in solid state environment. We theoretically found [2] that the leading channel of the $p+d \to {}^3He$ reaction in solid environment is the so called solid state internal conversion process, an adapted version of ordinary internal conversion process [3]. It was shown [2] that if the reaction $p+d \to {}^3He$ takes place in solid material the nuclear energy is taken away by an electron of the environment instead of the emission of a γ photon.

These observations raise the question of the possibility of further modification of nuclear processes due to solid state environment. In this paper the electron assisted neutron exchange process is discussed in solid state environment.

Let us consider the following general nuclear reaction

$$ {}^{A_1}_{Z_1}X + {}^{A_2}_{Z_2}Y \to {}^{A_1-1}_{Z_1}X + {}^{A_2+1}_{Z_2}Y + \Delta, \tag{1}$$

which is called neutron exchange reaction further on. Here Δ is the energy of the reaction, i.e. the difference between the rest energies of the initial $\left({}^{A_1}_{Z_1}X + {}^{A_2}_{Z_2}Y\right)$ and final $\left({}^{A_1-1}_{Z_1}X + {}^{A_2+1}_{Z_2}Y\right)$ states. In (1) the ${}^{A_1}_{Z_1}X$ nucleus loses a neutron which is taken up by the ${}^{A_2}_{Z_2}Y$ nucleus. The process is energetically forbidden if $\Delta < 0$. If the relative energy of the two initial nuclei is high enough to bring them within or near to the range of the nuclear force process (1) usually takes place spontaneously. It is also a process of type (1) if $Z_1 = Z_2 = Z$, i.e. the process

$$ {}^{A_1}_{Z}X + {}^{A_2}_{Z}X \to {}^{A_1-1}_{Z}X + {}^{A_2+1}_{Z}X + \Delta \tag{2}$$

which is considered further on. One possible realization of process (2) is if the beam consists of particles ${}^{A_1}_{Z}X$ and particles ${}^{A_2}_{Z}X$ are targets.

However, if the energy of the beam is less than the neutron separation energy then the Coulomb interaction between projectile and target nuclei creates a virtual neutron which is captured by the other nucleus. So in this case the process can be considered as a second order process from the point of view of perturbation calculation.

The cross section of process (2) can be derived applying the Coulomb solution $\varphi(\mathbf{r})$, which is the wave function of a free particle of charge number Z in a repulsive Coulomb field of charge number Z [4], in the description of relative motion of projectile and target. Since $\varphi(\mathbf{r}) \sim e^{-\pi\eta/2}\Gamma(1+i\eta)$, the cross section of the process is proportional to

$$\left|e^{-\pi\eta/2}\Gamma(1+i\eta)\right|^2 = \frac{2\pi\eta\left(E\right)}{\exp\left[2\pi\eta\left(E\right)\right]-1} = F_C(E). \tag{3}$$

Here Γ is the Gamma function and η is the Sommerfeld parameter which reads as

$$\eta\left(E\right) = Z^2\alpha_f\sqrt{\left(\frac{A_1A_2}{A_1+A_2}\right)\frac{m_0c^2}{2E}} \tag{4}$$

in the case of colliding particles of charge numbers $Z_1 = Z_2 = Z$ and rest masses $m_1 = A_1m_0$ and $m_2 = A_2m_0$. E is the kinetic energy in the center of mass coordinate system, $m_0c^2 = 931.494\ MeV$ is the atomic energy unit and α_f is the fine structure constant. Thus it is a fact that the rate of the nuclear reaction (2) becomes very small at low energies as a consequence of $F_C(E)$ being small.

The electron assisted version of process (2) is

$$e + {}^{A_1}_{Z}X + {}^{A_2}_{Z}X \rightarrow e' + {}^{A_1-1}_{Z}X + {}^{A_2+1}_{Z}X + \Delta \tag{5}$$

where e and e' denote electron. It is also a second order process in which the electron Coulomb interacts with the ${}^{A_1}_{Z}X$ nucleus, the intermediate, virtual neutron and the ${}^{A_1-1}_{Z}X$ nucleus are created due to this interaction and the intermediate, virtual neutron is captured due to the strong interaction by the nucleus ${}^{A_2}_{Z}X$ forming the nucleus ${}^{A_2+1}_{Z}X$ in this manner.

The physical background of the virtual neutron stripping due to the Coulomb interaction is worth mentioning. The attractive Coulomb interaction acts between the Z protons and the electron. The neutrons do not feel Coulomb interaction. So one can say that in fact the nucleus ${}^{A_1-1}_{Z}X$ is stripped of the neutron due to the Coulomb attraction.

When describing the effect of the Coulomb interaction between the nucleus of charge number Z and a slow electron one can also use Coulomb function and, consequently, the cross section of process (5) to be investigated is proportional to

$$F_e(E) = \frac{2\pi\eta_e\left(E\right)}{\exp\left[2\pi\eta_e\left(E\right)\right]-1} \tag{6}$$

but with

$$\eta_e = -Z\alpha_f\sqrt{\frac{m_ec^2}{2E}}. \tag{7}$$

Here m_e is the rest mass of the electron. In the case of low (less than 0.1 MeV) kinetic energy of the electron $F_e(E)$ reads approximately as $F_e(E) = |2\pi\eta_e(E)|$. If we compare the cross sections of processes (5) and (2) their ratio is proportional to $F_e(E)/F_C(E) \simeq 1/F_C(E) \gg 1$. Therefore for small E process (5) is preferred to process (2) since $1/F_C(E)$ becomes extremely large with decreasing E. The cross section of the electron assisted neutron exchange process has a further increase due to the large number density $\simeq 1/d^3$ of the two types of nuclei in the solid to which the cross section is also proportional as it will be seen. Here d is the lattice parameter of order of magnitude of 10^{-8} cm.

We investigate systems in which process (5) can take place. The solid is a metal (e.g. Ni or Pd) which contains the nuclei ${}^{A_1}_{Z}X$ and ${}^{A_2}_{Z}X$ and it is irradiated with slow, free electrons (of nonrelativistic energy). The nuclei ${}^{A_1}_{Z}X$ (or ${}^{A_2}_{Z}X$) interact with the ingoing free electrons via Coulomb interaction.

II. ELECTRON ASSISTED NEUTRON EXCHANGE PROCESS

Let us take a solid (in our case a metal) which is irradiated by a monoenergetic beam of slow, free electrons. The corresponding sub-system Hamiltonians are H_{solid} and H_e. It is supposed that their eigenvalue problems are solved, and the complete set of the eigenvectors of the two independent systems are known. The interaction between them is the Coulomb interaction of potential $V^{Cb}(\mathbf{x})$ and the other interaction that is taken into account between the nucleons of the solid is the strong interaction potential $V^{St}(\mathbf{x})$. In the second order process investigated an electron takes part in a Coulomb scattering with an atomic nucleus of the solid. In the intermediate state a virtual free neutron n is created which is captured due to the strong interaction with some other nucleus of the solid. The reaction energy Δ is shared between the quasi-free final electron and the two final nuclei which take part in the process. Since the aim of this paper is to show the fundamentals of the main effect, the simplest description is chosen.

The electron of charge $-e$ and the nucleus ${}^{A_1}_{Z}X$ of charge Ze take part in Coulomb-interaction. We use a screened Coulomb potential of the form

$$V^{Cb}(\mathbf{x}) = \int \frac{-4\pi e^2 Z}{q^2 + \lambda^2} \exp(i\mathbf{q}\cdot\mathbf{x})\, d\mathbf{q} \tag{8}$$

with screening parameter λ and coupling strength $e^2 = \alpha_f \hbar c$. For the strong interaction the

interaction potential

$$V^{St}(\mathbf{x}) = -f\frac{\exp(-s|\mathbf{x}|)}{|\mathbf{x}|} \tag{9}$$

is applied, where the strong coupling strength $f = 0.08\hbar c$ [5] and $1/s$ is the range of the strong interaction. ($\hbar$ is the reduced Planck constant, c is the velocity of light and e is the elementary charge.)

According to the standard perturbation theory of quantum mechanics the transition probability per unit time (W_{fi}) of this second order process can be written as

$$W_{fi} = \frac{2\pi}{\hbar}\sum_f |T_{fi}|^2 \delta(E_f - E_i - \Delta) \tag{10}$$

with

$$T_{fi} = \sum_\mu \frac{V^{St}_{f\mu} V^{Cb}_{\mu i}}{\Delta E_{\mu i}}. \tag{11}$$

Here $V^{Cb}_{\mu i}$ is the matrix element of the Coulomb potential between the initial and intermediate states and $V^{St}_{f\mu}$ is the matrix element of the potential of the strong interaction between intermediate and final states, furthermore

$$\Delta E_{\mu i} = E_\mu - E_i - \Delta_{i\mu}. \tag{12}$$

E_i, E_μ and E_f are the kinetic energies in the initial, intermediate and final states, respectively, Δ is the reaction energy, and $\Delta_{i\mu}$ is the difference between the rest energies of the initial $\left({}^{A_1}_{Z}X\right)$ and intermediate $\left({}^{A_1-1}_{Z}X \text{ and } n\right)$ states.

$$\Delta = \Delta_- + \Delta_+,\ \Delta_{i\mu} = \Delta_- - \Delta_n \tag{13}$$

with

$$\Delta_- = \Delta_{A_1} - \Delta_{A_1-1} \text{ and } \Delta_+ = \Delta_{A_2} - \Delta_{A_2+1}. \tag{14}$$

Δ_{A_1}, Δ_{A_1-1}, Δ_{A_2}, Δ_{A_2+1} and Δ_n are the energy excesses of the neutral atoms of mass numbers A_1, A_1-1, A_2, A_2+1 and the neutron, respectively. [6]. The sum of initial kinetic energies (E_i) is neglected in the energy Dirac-delta $\delta(E_f - E_i - \Delta)$ and $\Delta E_{\mu i}$ further on.

Particle e is an electron, particle 1 is initially the nucleus ${}^{A_1}_{Z}X$ and finally ${}^{A_1-1}_{Z}X$, particle 2 is initially the nucleus ${}^{A_2}_{Z}X$ and finally ${}^{A_2+1}_{Z}X$.

$$E_f = E_{fe}(\mathbf{k}_{fe}) + E_{f1}(\mathbf{k}_1) + E_{f2}(\mathbf{k}_2), \tag{15}$$

$$E_{\mu} = E_{fe}\left(\mathbf{k}_{fe}\right) + E_{\mu 1}\left(\mathbf{k}_1\right) + E_n\left(\mathbf{k}_n\right), \tag{16}$$

where

$$E_{fj}\left(\mathbf{k}_j\right) = \frac{\hbar^2 \mathbf{k}_j^2}{2m_j} \tag{17}$$

is the kinetic energy, $\mathbf{k}_{fj} \equiv \mathbf{k}_j$ is the wave vector and m_j is the rest mass of particle j in the final state ($j = 1, 2$).

$$E_n\left(\mathbf{k}_n\right) = \frac{\hbar^2 \mathbf{k}_n^2}{2m_n} \tag{18}$$

is the kinetic energy, $\mathbf{k}_n$ is the wave vector and m_n is the rest mass of the neutron in the intermediate state. $E_{\mu 1}\left(\mathbf{k}_1\right)$ is the kinetic energy of the first particle in the intermediate state, and $E_{\mu 1}\left(\mathbf{k}_1\right) = E_{f1}\left(\mathbf{k}_1\right)$. The kinetic energy of the electron in the initial and final state

$$E_{ie} = \frac{\hbar^2 \mathbf{k}_{ie}^2}{2m_e} \text{ and } E_{fe} = \frac{\hbar^2 \mathbf{k}_{fe}^2}{2m_e} \tag{19}$$

with $\mathbf{k}_{ie}$ and $\mathbf{k}_{fe}$ denoting the wave vector of the electron in the initial and final state. The initial wave vectors $\mathbf{k}_{i1}$ and $\mathbf{k}_{i2}$ of particles 1 and 2 are neglected. The initial, intermediate and final states are determined in Appendix A., the $V_{\mu i}^{Cb}$, $V_{f\mu}^{St}$ matrix-elements are calculated in Appendix B. and the transition probability per unit time is calculated in Appendix C.. Appendix D. is devoted to the approximations, identities and relations which are used in the calculation of the cross section.

III. CROSS SECTION AND YIELD OF EVENTS OF ELECTRON ASSISTED NEUTRON EXCHANGE PROCESS

A. Cross section of electron assisted neutron exchange process

The cross section σ of the process can be obtained from the transition probability per unit time (56) dividing it by the flux v_e/V of the incoming electron where v_e is the velocity of the electron.

$$\begin{aligned} \sigma \;=\; & \int \frac{c}{v_e} \frac{64\pi^3 \alpha_f^2 \hbar c Z^2 \sum_{l_2=-m_2}^{l_2=m_2} \left|F_2\left(\mathbf{k}_2\right)\right|^2}{v_c \left(\left|\mathbf{k}_1 + \mathbf{k}_2\right|^2 + \lambda^2\right)^2 \left(\Delta E_{\mu i}\right)_{\mathbf{k}_n = \mathbf{k}_2}^2} \\ & \times \frac{F_e(E_{ie})}{F_e(E_{f1})} \left\langle \left|F_1\left(\mathbf{k}_2\right)\right|^2 \right\rangle A_2^2 r_{A_2} \delta(E_f - \Delta) d^3k_1 d^3k_2, \end{aligned} \tag{20}$$

where v_c is the volume of elementary cell in the solid, r_{A_2} is the relative natural abundance of atoms ${}^{A_2}_{Z}X$,

$$F_1(\mathbf{k}_2) = \int \Phi_{i1}(\mathbf{r}_{n1})\, e^{-i\mathbf{k}_2 \frac{A_1}{A_1-1}\cdot \mathbf{r}_{n1}} d^3 r_{n1}, \tag{21}$$

$$\left\langle |F_1(\mathbf{k}_2)|^2 \right\rangle = \frac{1}{2l_1+1} \sum_{l_1=-m_1}^{l_1=m_1} |F_1(\mathbf{k}_2)|^2 \tag{22}$$

and

$$\begin{aligned} F_2(\mathbf{k}_2) \;=\; & \int \Phi^*_{f2}(\mathbf{r}_{n2})\, e^{i\mathbf{k}_2\cdot\mathbf{r}_{n2}} \times \\ & \times \left(-f \frac{\exp(-s\frac{A_2+1}{A_2} r_{n2}}{\frac{A_2+1}{A_2} r_{n2}} \right) d^3 r_{n2}. \end{aligned} \tag{23}$$

Here Φ_{i1} and Φ_{f2} are the initial and final bound neutron states. The details of the cross section calculation (see in Appendix D.) result that the $k_2 \simeq k_0 = \sqrt{2\mu_{12}\Delta}/\hbar$ substitution may be used in calculating F_1 and F_2 in σ, where $\mu_{12} = m_0\left[(A_1-1)(A_2+1)\right]/(A_1+A_2)$.

Evaluating (20) first the Weisskopf approximation is applied, i.e. for the initial and final bound neutron states we take $\Phi_W(\mathbf{r}_{nj}) = \phi(r_{nj})\, Y_{l_j m_j}(\Omega_j)$, $j = 1,2$ where $Y_{l_j m_j}(\Omega_j)$ is a spherical harmonics and $\phi_{jW}(r_{nj}) = \sqrt{3/R_j^3}$, $j = 1,2$ if $|\mathbf{r}_{nj}| \le R_j$ and $\phi_{jW}(r_{nj}) = 0$ for $|\mathbf{r}_{nj}| > R_j$, where $R_j = r_0 A_j^{1/3}$is the radius of a nucleus of nucleon number A_j with $r_0 = 1.2\times 10^{-13}\ cm$. We apply the $A_1 \simeq A_2 \simeq A_1 - 1 \simeq A_2 + 1 = A$ approximation further on. Calculating $F_1(\mathbf{k}_0)$ and $F_2(\mathbf{k}_0)$ the long wavelength approximations (LWA) ($\exp(-i\mathbf{k}_0\cdot\mathbf{r}_{n1}) = 1$ and $\exp(i\mathbf{k}_0\cdot\mathbf{r}_{n2}) = 1$) are also used with $s = 1/r_0$ that result approximately

$$\left\langle |F_1(\mathbf{k}_0)|^2 \right\rangle \sum_{l_2=-m_2}^{l_2=m_2} |F_2(\mathbf{k}_0)|^2 = 16\pi^2 r_0^4 f^2 (2l_2+1). \tag{24}$$

Using the results of Appendix D., the $E_{f1} = \Delta/2$ relation and if $E_e < 0.1\ MeV$ (i.e. if $F_e(E_{ie}) = |2\pi\eta_e(E_{ie})| = 2\pi Z\alpha_f\sqrt{m_e c^2/2E_{ie}}$) then the cross section in the Weisskopf-LWA approximation reads as

$$\sigma_W = \frac{C_{W0}(2l_2+1)}{\left[1+\frac{2(\Delta_n-\Delta_-)}{A\Delta}\right]^2} \frac{r_{A_2}}{F_e(\Delta/2)} \frac{A^{3/2}Z^2}{\Delta^{3/2}E_{ie}} \tag{25}$$

with $C_{W0} = 2^{15}\pi^9\alpha_f^3 (0.08)^2 a_B r_0 \left(\frac{r_0}{d}\right)^3 (m_0c^2)^{3/2} m_e c^2$. Here a_B is the Bohr-radius, the relation $c/v_e = \sqrt{m_e c^2/(2E_{ie})}$ with E_{ie} the kinetic energy of the ingoing electrons is also applied and $d = 3.52\times 10^{-8}\ cm$ (Ni lattice) and $d = 3.89\times 10^{-8}\ cm$ (Pd lattice). $F_e(\Delta/2)$ is determined by (6) and (7). The subscript W refers to the Weisskopf-LWA approximation and in

(25) the quantities Δ and E_{ie} have to be substituted in MeV units. $C_{W0}(Ni) = 8.9 \times 10^{-10}$ $MeV^{5/2}b$ and $C_{W0}(Pd) = 6.6 \times 10^{-10}$ $MeV^{5/2}b$.

We have calculated $\sum_{l_2=-m_2}^{l_2=m_2} |F_2(\mathbf{k}_0)|^2$, $\left\langle |F_1(\mathbf{k}_0)|^2 \right\rangle$ and the cross section in the single particle shell model with isotropic harmonic oscillator potential and without the long wavelength approximation (see Appendix E.). We introduce the ratio

$$\eta = \frac{\left\langle |F_1(\mathbf{k}_0)|^2 \right\rangle_{Sh} \sum_{l_2=-m_2}^{l_2=m_2} |F_2(\mathbf{k}_0)|_{Sh}^2}{\left\langle |F_1(\mathbf{k}_0)|^2 \right\rangle_{W} \sum_{l_2=-m_2}^{l_2=m_2} |F_2(\mathbf{k}_0)|_{W}^2}. \tag{26}$$

(The subscript Sh refers to the shell model.) With the aid of $\eta \equiv \eta_{l_1,n_1,l_2,n_2}(A_1, A_2)$ given by (74) (see Appendix E.) the cross section σ_{Sh} calculated in the shell model can be written as

$$\sigma_{Sh} = \eta_{l_1,n_1,l_2,n_2}(A_1, A_2)\, \sigma_W. \tag{27}$$

B. Yield of events of electron assisted neutron exchange process

The yield dN/dt of events of electron assisted neutron exchange process $A_1, A_2 \to A_1 - 1, A_2 + 1$ can be written as

$$\frac{dN}{dt} = N_t N_{ni} \sigma \Phi, \tag{28}$$

where $\sigma = \{\sigma_W \text{ or } \sigma_{Sh}\}$, Φ is the flux of electrons, N_t is the number of target particles, i.e. the number N_{A_1} of irradiated atoms of mass number A_1 in the metal. The contribution of N_{ni} neutrons in each nucleus ${}_{Z}^{A_1}X$ is also taken into account. N_{ni} is the number of neutrons in the uppermost energy level of the initial nucleus ${}_{Z}^{A_1}X$. If F and D are the irradiated surface and the width of the sample, respectively, then the number of elementary cells N_c in the sample is $N_c = FD/v_c = 4FD/d^3$ in the case of Ni and Pd, and the number of atoms in the elementary cell is $2r_{A_1}$ with r_{A_1} the relative natural abundance of atoms ${}_{Z}^{A_1}X$ thus the number N_t of target atoms of mass number A_1 in the process is

$$N_t = \frac{8}{d^3} r_{A_1} FD. \tag{29}$$

The wave numbers and energies of the two outgoing heavy particles are approximately $\mathbf{k}_1 = -\mathbf{k}_2$,

$$E_1 = \frac{A_2 + 1}{A_1 + A_2}\Delta \text{ and } E_2 = \frac{A_1 - 1}{A_1 + A_2}\Delta. \tag{30}$$

A	58	60	61	62	64
Δ_-	-4.147	-3.317	0.251	-2.526	-1.587
Δ_+	0.928	-0.251	2.526	-1.234	-1.973
r_A	0.68077	0.26223	0.0114	0.03634	0.00926

TABLE I: Numerical data of the $e + {}^{A_1}_{28}Ni + {}^{A_2}_{28}Ni \to e' + {}^{A_1-1}_{28}Ni + {}^{A_2+1}_{28}Ni + \Delta$ reaction. The reaction is energetically allowed if $\Delta = \Delta_-(A_1) + \Delta_+(A_2) > 0$ holds. A is the mass number, r_A is the relative natural abundance, $\Delta_-(A) = \Delta_A - \Delta_{A-1}$ and $\Delta_+(A) = \Delta_A - \Delta_{A+1}$ are given in MeV units.

A	102	104	105	106	108	110
Δ_-	-2.497	-1.912	0.978	-1.491	-1.149	-0.747
Δ_+	-0.446	-0.978	1.491	-1.533	-1.918	-2.320
r_A	0.0102	0.1114	0.2233	0.2733	0.2646	0.1172

TABLE II: Numerical data of the $e + {}^{A_1}_{46}Pd + {}^{A_2}_{46}Pd \to e' + {}^{A_1-1}_{46}Pd + {}^{A_2+1}_{46}Pd + \Delta$ reaction. The reaction is energetically allowed if $\Delta = \Delta_-(A_1) + \Delta_+(A_2) > 0$ holds. A is the mass number, r_A is the relative natural abundance, $\Delta_-(A) = \Delta_A - \Delta_{A-1}$ and $\Delta_+(A) = \Delta_A - \Delta_{A+1}$ are given in MeV units.

IV. NUMERICAL DATA OF ELECTRON ASSISTED NEUTRON EXCHANGE PROCESSES IN Ni AND Pd

As a first example we take Ni as target material. In this case the possible processes are

$$e + {}^{A_1}_{28}Ni + {}^{A_2}_{28}Ni \to e' + {}^{A_1-1}_{28}Ni + {}^{A_2+1}_{28}Ni + \Delta. \tag{31}$$

Tables I. and III. contain the relevant data for reaction (31). Describing neutrons in the uppermost energy level of ${}^{A}_{28}Ni$ isotopes we used $1p$ shell model states in the cases of $A = 58 - 60$ and $0f$ shell model states in the cases of $A = 61 - 64$.

Another interesting target material is Pd in which the electron assisted neutron exchange processes are the

$$e + {}^{A_1}_{46}Pd + {}^{A_2}_{46}Pd \to e' + {}^{A_1-1}_{46}Pd + {}^{A_2+1}_{46}Pd + \Delta \tag{32}$$

$A_1 \to A_1 - 1$	$A_2 \to A_2 + 1$	Δ(MeV)	η
$61 \to 60$	$58 \to 59$	1.179	7.02×10^{-3}
$61 \to 60$	$61 \to 62$	2.777	2.42×10^{-8}
$64 \to 63$	$61 \to 62$	0.939	2.08×10^{-4}

TABLE III: The values of the quantities η and $\Delta = \Delta_-(A_1) + \Delta_+(A_2) > 0$, the later in MeV units, of the $e + {}^{A_1}_{28}Ni + {}^{A_2}_{28}Ni \to e' + {}^{A_1-1}_{28}Ni + {}^{A_2+1}_{28}Ni + \Delta$ reaction. The $\Delta_-(A_1)$ and $\Delta_+(A_2)$ values can be found in Table I. For the definition of η see (26) and (74).

$A_1 \to A_1 - 1$	$A_2 \to A_2 + 1$	Δ(MeV)	η
$105 \to 104$	$102 \to 103$	0.532	1.84×10^{-4}
$105 \to 104$	$105 \to 106$	2.469	8.88×10^{-11}
$108 \to 107$	$105 \to 106$	0.342	2.82×10^{-3}

TABLE IV: The values of the quantities η and $\Delta = \Delta_-(A_1) + \Delta_+(A_2) > 0$, the later in MeV units, of the $e + {}^{A_1}_{46}Pd + {}^{A_2}_{46}Pd \to e' + {}^{A_1-1}_{46}Pd + {}^{A_2+1}_{46}Pd + \Delta$ reaction. The $\Delta_-(A_1)$ and $\Delta_+(A_2)$ values can be found in Table II. For the definition of η see (26) and (74).

reactions. The relevant data can be found in Tables II. and IV.. Describing neutrons in the uppermost energy level of ${}^{A}_{46}Pd$ isotopes we used $0g$ shell model states in the cases of $A = 102 - 104$ and $1d$ shell model states in the cases of $A = 105 - 108$. The nuclear data to the Tables are taken from [6]. One can see from Tables III. and IV. that in both cases three possible pairs of isotopes exist which are energetically allowed (for which $\Delta > 0$) and their rates differ in the factor $(2l_2 + 1)\, N_{ni} \eta_{l_1,n_1,l_2,n_2}(A_1, A_2)\, r_{A_1} r_{A_2} \Delta^{-3/2}$ only. The $\eta \equiv \eta_{l_1,n_1,l_2,n_2}(A_1, A_2)$ values of Ni and Pd can also be found in Tables III. and IV., respectively. The results of numerical investigation of $(2l_2 + 1)\, N_{ni} \eta_{l_1,n_1,l_2,n_2}(A_1, A_2)\, r_{A_1} r_{A_2} \Delta^{-3/2}$ shows that the $61 \to 60, 58 \to 59$ and the $108 \to 107, 105 \to 106$ reactions are the dominant among the processes in Ni and Pd, respectively.

V. SUMMARY

The electron assisted neutron exchange process is discussed. The transition probability per unit time and the cross section of the process are determined in those cases when the electron takes off negligible energy. The electron assisted neutron exchange processes are investigated numerically in Ni and Pd. In the case of Ni it is found that the

$$e + {}^{61}_{28}Ni + {}^{58}_{28}Ni \rightarrow e' + {}^{60}_{28}Ni + {}^{59}_{28}Ni + 1.179\ MeV \tag{33}$$

process of $\sigma_{Sh} = 0.54/E_{ie}\ mb$ with E_{ie} in MeV is leading. In this case the ${}^{60}_{28}Ni$ and the ${}^{59}_{28}Ni$ isotopes take away $0.585\ MeV$ and $0.594\ MeV$, respectively. In the case of Pd the

$$e + {}^{108}_{46}Pd + {}^{105}_{46}Pd \rightarrow e' + {}^{107}_{46}Pd + {}^{106}_{46}Pd + 0.342\ MeV \tag{34}$$

reaction of $\sigma_{Sh} = 1.6/E_{ie}\ mb$ with E_{ie} in MeV is found to be the leading one. In this case the ${}^{107}_{46}Pd$ and the ${}^{106}_{46}Pd$ isotopes take away $0.170\ MeV$ and $0.172\ MeV$, respectively.

There are many other materials which may be suitable for hosting electron assisted neutron exchange process some of which we list in the following. We deal with period 4 of transition metals. (The metal targets are advantageous from experimental point of view since in the case of metals it is easy to avoid the charging of the sample.) The Cr, Fe and V have body centered cubic crystal lattice. In the case of V there is one and in the case of Cr and Fe there are two energetically allowed electron assisted neutron exchange processes. Ti and Zn have closepacked hexagonal crystal structure. In the case of Ti there are five and in the case of Zn there are three energetically allowed electron assisted neutron exchange processes. Since the natural abundance of ${}^{45}_{25}Sc$, ${}^{55}_{25}Mn$ and ${}^{59}_{27}Co$ equals unity in the case of these materials there is no chance of the electron assisted neutron exchange process. Although Cu has two natural isotopes, their electron assisted neutron exchange processes are energetically forbidden.

VI. APPENDIX

A. Initial, intermediate and final states of the process

Let Ψ_i, Ψ_μ and Ψ_f denote the space dependent parts of initial, intermediate and final states, respectively. The initial state has the form

$$\Psi_i(\mathbf{x}_e, \mathbf{x}_1, \mathbf{x}_{n1}, \mathbf{x}_2) = \psi_{ie}(\mathbf{x}_e)\, \psi_{i1n}(\mathbf{x}_1, \mathbf{x}_{n1})\psi_{i2}(\mathbf{x}_2), \tag{35}$$

where

$$\psi_{ie}(\mathbf{x}_e) = V^{-1/2} e^{(i\mathbf{k}_{ie}\cdot\mathbf{x}_e)} \text{ and } \psi_{i2}(\mathbf{x}_2) = V^{-1/2} e^{(i\mathbf{k}_{i2}\cdot\mathbf{x}_2)} \tag{36}$$

are the initial state of the electron and the nucleus ${}^{A_2}_{Z}X$, and $\psi_{i1n}(\mathbf{x}_1, \mathbf{x}_{n1})$ is the initial state of the neutron and the initial $A_1 - 1$ nucleon of the nucleus ${}^{A_1}_{Z}X$. $\mathbf{x}_e$, $\mathbf{x}_1, \mathbf{x}_{n1}$ and $\mathbf{x}_2$ are the coordinates of the electron, the center of mass of the initial $A_1 - 1$ nucleon, the neutron and the nucleus ${}^{A_2}_{Z}X$, respectively. $\mathbf{k}_{ie}$ and $\mathbf{k}_{i2}$ are the initial wave vectors of the electron and the nucleus ${}^{A_2}_{Z}X$ and V is the volume of normalization. The initial state $\psi_{i1n}(\mathbf{x}_1, \mathbf{x}_{n1})$ of the neutron and the initial $A_1 - 1$ nucleon may be given in the variables $\mathbf{R}_1$, $\mathbf{r}_{n1}$

$$\psi_{i1n}(\mathbf{R}_1, \mathbf{r}_{n1}) = V^{-1/2} \exp(i\mathbf{k}_{i1}\cdot\mathbf{R}_1)\Phi_{i1}(\mathbf{r}_{n1}) \tag{37}$$

where $\mathbf{R}_1$ is the center of mass coordinate of the nucleus ${}^{A_1}_{Z}X$ and $\mathbf{r}_{n1}$ is the relative coordinate of one of its neutrons. $\mathbf{R}_1$ and $\mathbf{r}_{n1}$are determined by the usual $\mathbf{x}_{n1} = \mathbf{R}_1 + \mathbf{r}_{n1}$ and $\mathbf{R}_1 = [(A_1 - 1)\mathbf{x}_1 + \mathbf{x}_{n1}]/A_1$ relations where $\mathbf{x}_{n1}$ and $\mathbf{x}_1$ are the coordinates of the neutron and of the center of mass of the initial $A_1 - 1$ nucleon, respectively. The inverse formula for $\mathbf{x}_1$ is $\mathbf{x}_1 = \mathbf{R}_1 - \mathbf{r}_{n1}/(A_1 - 1)$. In (37) the $\Phi_{i1}(\mathbf{r}_{n1})$ is the wave function of the neutron in the initial bound state of nucleus ${}^{A_1}_{Z}X$, $\mathbf{k}_{i1}$is the initial wave vector of nucleus ${}^{A_1}_{Z}X$.

The intermediate state has the form

$$\Psi_\mu(\mathbf{x}_e, \mathbf{x}_1, \mathbf{x}_{n1}, \mathbf{x}_2) = \psi_{fe}(\mathbf{x}_e)\, \psi_{\mu 1n}(\mathbf{x}_1, \mathbf{x}_{n1})\psi_{i2}(\mathbf{x}_2), \tag{38}$$

where

$$\psi_{fe}(\mathbf{x}_e) = V^{-1/2} e^{\left(i\mathbf{k}_{fe}\cdot\mathbf{x}_e\right)} \tag{39}$$

with $\mathbf{k}_{fe}$ the wave vector of the electron in the final state and $\psi_{i2}(\mathbf{x}_2)$ is given in (36). The state $\psi_{\mu 1n}(\mathbf{x}_1, \mathbf{x}_{n1})$ is the product of two plane waves $\psi_{f1}(\mathbf{x}_1) = V^{-1/2} e^{(i\mathbf{k}_1\cdot\mathbf{x}_1)}$ and $\psi_n(\mathbf{x}_{n1}) = V^{-1/2} e^{i\mathbf{k}_n\cdot\mathbf{x}_{n1}}$, which are the final state of the nucleus ${}^{A_1-1}_{Z_1}X$ and the state of the

free, intermediate neutron. Thus $\psi_{\mu 1n}(\mathbf{x}_1, \mathbf{x}_{n1}) = V^{-1} e^{i\mathbf{k}_1 \cdot \mathbf{x}_1} e^{i\mathbf{k}_n \cdot \mathbf{x}_{n1}}$ and it has the form in the coordinates $\mathbf{R}_1$, $\mathbf{r}_{n1}$

$$\psi_{\mu 1n}(\mathbf{R}_1, \mathbf{r}_{n1}) = V^{-1} e^{i(\mathbf{k}_1 + \mathbf{k}_n)\cdot \mathbf{R}_1} e^{i\left(\mathbf{k}_n - \frac{\mathbf{k}_1}{A_1 - 1}\right)\mathbf{r}_{n1}}, \tag{40}$$

where $\mathbf{k}_1$ and $\mathbf{k}_n$ are the wave vectors of the nucleus ${}^{A_1-1}_{Z}X$ and the neutron, respectively.

The intermediate state may have an other form

$$\Psi_\mu(\mathbf{x}_e, \mathbf{x}_1, \mathbf{x}_{n1}, \mathbf{x}_2) = \psi_{fe}(\mathbf{x}_e)\, \psi_{f1}(\mathbf{x}_1) \psi_{\mu 2n}(\mathbf{x}_{n1}, \mathbf{x}_2), \tag{41}$$

where

$$\psi_{\mu 2n}(\mathbf{x}_{n1}, \mathbf{x}_2) = \psi_n(\mathbf{x}_{n1})\, \psi_{i2}(\mathbf{x}_2) = V^{-1} e^{i\mathbf{k}_n \cdot \mathbf{x}_{n1}} e^{i\mathbf{k}_{i2} \cdot \mathbf{x}_2} \tag{42}$$

which can be written in the coordinates $\mathbf{r}_{n2} = \mathbf{x}_{n1} - \mathbf{R}_2$ and $\mathbf{R}_2 = (A_2 \mathbf{x}_2 + \mathbf{x}_{n1}) / (A_2 + 1)$ as

$$\psi_{\mu 2n}(\mathbf{R}_2, \mathbf{r}_{n2}) = \frac{1}{V} e^{i(\mathbf{k}_{i2} + \mathbf{k}_n)\cdot \mathbf{R}_2} e^{i\left(\mathbf{k}_n - \frac{\mathbf{k}_{i2}}{A_2}\right)\mathbf{r}_{n2}}, \tag{43}$$

where $\mathbf{R}_2$ is the center of mass coordinate of the nucleus ${}^{A_2+1}_{Z}X$ and $\mathbf{r}_{n2}$ is the relative coordinate of the neutron in it. In these new variables $\mathbf{x}_2 = \mathbf{R}_2 - \mathbf{r}_{n2}/A_2$ and $\mathbf{x}_{n1} - \mathbf{x}_2 = (A_2 + 1)\, \mathbf{r}_{n2}/A_2$ which is used in the argument of V^{St} (given by (9)) in calculating $V^{St}_{f\mu}$. Evaluating the matrix elements $V^{Cb}_{\mu i}$ and $V^{St}_{f\mu}$ the forms (40) and (43) of ψ_μ are used, respectively, and $\sum_\mu \to \frac{V}{(2\pi)^3} d^3 k_n$ in (11).

The final state has the form

$$\Psi_f(\mathbf{x}_e, \mathbf{x}_1, \mathbf{x}_{n1}, \mathbf{x}_2) = \psi_{fe}(\mathbf{x}_e)\, \psi_{f1}(\mathbf{x}_1) \psi_{f2n}(\mathbf{x}_{n1}, \mathbf{x}_2), \tag{44}$$

where $\psi_{f2n}(\mathbf{x}_{n1}, \mathbf{x}_2)$ is given in the variables $\mathbf{R}_2$, $\mathbf{r}_{n2}$ as

$$\psi_{f2n}(\mathbf{R}_2, \mathbf{r}_{n2}) = V^{-1/2} \exp(i\mathbf{k}_2 \cdot \mathbf{R}_2) \Phi_{f2}(\mathbf{r}_{n2}), \tag{45}$$

and $\Phi_{f2}(\mathbf{r}_{n2})$ is the bound state of the neutron in the nucleus ${}^{A_2+1}_{Z}X$.

B. Evaluation of matrix elements $V^{Cb}_{\mu i}$ and $V^{St}_{f\mu}$

The argument of the Coulomb potential V^{Cb} is $\mathbf{x}_e - \mathbf{x}_1$ therefore the integration with respect to the components of $\mathbf{x}_2$ may be carried out and $\int |\psi_{i2}(\mathbf{x}_2)|^2 d^3 x_2 = 1$. The remainder is

$$\begin{aligned} V^{Cb}_{\mu i} \;=\; & \int \psi^*_{fe}(\mathbf{x}_e)\, \psi^*_{\mu 1n}(\mathbf{x}_1, \mathbf{x}_{n1}) V^{Cb}(\mathbf{x}_e - \mathbf{x}_1) \\ & \times \psi_{ie}(\mathbf{x}_e)\, \psi_{i1n}(\mathbf{x}_1, \mathbf{x}_{n1}) d^3 x_e d^3 x_1 d^3 x_{n1}. \end{aligned} \tag{46}$$

Making the $\mathbf{x}_1, \mathbf{x}_{n1} \to \mathbf{R}_1, \mathbf{r}_{n1}$ change in the variables, substituting the forms (37) and (40) of ψ_{i1n} and $\psi_{\mu 1n}$, and neglecting $\mathbf{k}_{i1}$, the integrations over the components of $\mathbf{x}_e$ and $\mathbf{R}_1$ result $V^{-1}(2\pi)^3 \delta(\mathbf{q} + \mathbf{k}_{ie} - \mathbf{k}_{fe})$ and $V^{-3/2}(2\pi)^3 \delta(\mathbf{q} - \mathbf{k}_1 - \mathbf{k}_n)$, respectively and the integration over the components of $\mathbf{r}_{n1}$ produces $F_1(\mathbf{k}_n)$ where

$$F_1(\mathbf{k}_n) = \int \Phi_{i1}(\mathbf{r}_{n1}) e^{-i\left(\mathbf{k}_n - \frac{\mathbf{k}_1 + \mathbf{q}}{A_1 - 1}\right)\cdot \mathbf{r}_{n1}} d^3 r_{n1}. \tag{47}$$

Using the $\delta(\mathbf{q} + \mathbf{k}_{ie} - \mathbf{k}_{fe})$ in carrying out the integration over the components of $\mathbf{q}$ in $V_{\mu i}^{Cb}$ one gets

$$\begin{aligned} V_{\mu i}^{Cb} &= -\frac{4\pi e^2 Z}{|\mathbf{k}_{fe} - \mathbf{k}_{ie}|^2 + \lambda^2} \widetilde{F}_1(\mathbf{k}_n) \frac{(2\pi)^6}{V^{5/2}} \times \\ &\times \sqrt{G_S} \delta(\mathbf{k}_{ie} - \mathbf{k}_{fe} - \mathbf{k}_1 - \mathbf{k}_n) \end{aligned} \tag{48}$$

and

$$\widetilde{F}_1(\mathbf{k}_n) = \int \Phi_{i1}(\mathbf{r}_{n1}) e^{-i\left(\mathbf{k}_n - \frac{\mathbf{k}_1 + \mathbf{k}_{fe} - \mathbf{k}_{ie}}{A_1 - 1}\right)\cdot \mathbf{r}_{n1}} d^3 r_{n1}. \tag{49}$$

For particles e and 1 (ingoing electron of charge $-e$ and initial nucleus ${}_Z^{A_1}X$ of charge Ze) taking part in Coulomb interaction we have used plane waves therefore the matrix element must be corrected with the so called Sommerfeld factor [7] $\sqrt{G_S}$ where

$$G_S = \frac{F_e(E_{ie})}{F_e(E_{f1})}. \tag{50}$$

Now we deal with $V_{f\mu}^{St}$. The strong interaction works between the neutron and the nucleons of the nucleus ${}_Z^{A_2}X$ therefore the argument of V^{St} is $\mathbf{x}_{n1} - \mathbf{x}_2$. The integrations with respect to the components of $\mathbf{x}_e$ and $\mathbf{x}_1$ result $\int |\psi_{ef}(\mathbf{x}_e)|^2 d^3 x_e = \int |\psi_{f1}(\mathbf{x}_1)|^2 d^3 x_1 = 1$. The remainder is

$$V_{f\mu}^{St} = \int \psi_{f2n}^* V^{St}(\mathbf{x}_{n1} - \mathbf{x}_2) \psi_{\mu 2n} d^3 x_2 d^3 x_{n1}. \tag{51}$$

Similarly to the above, making the $\mathbf{x}_{n1}, \mathbf{x}_2 \to \mathbf{R}_2, \mathbf{r}_{n2}$ change in the variables, substituting the forms (43) and (45) of $\psi_{\mu 2n}$ and ψ_{f2n}^* and neglecting $\mathbf{k}_{i2}$, the integrations over the components of $\mathbf{R}_2$ result $V^{-3/2}(2\pi)^3 \delta(\mathbf{k}_n - \mathbf{k}_2)$ and the integrations with respect to the components of $\mathbf{r}_{n2}$ produces $F_2(\mathbf{k}_n)$ with

$$\begin{aligned} F_2(\mathbf{k}_n) &= \int \Phi_{f2}^*(\mathbf{r}_{n2}) e^{i\mathbf{k}_n \cdot \mathbf{r}_{n2}} \times \\ &\times \left(-f \frac{\exp(-s\frac{A_2+1}{A_2} r_{n2}}{\frac{A_2+1}{A_2} r_{n2}} \right) d^3 r_{n2}, \end{aligned} \tag{52}$$

where $r_{n2} = |\mathbf{r}_{n2}|$. Taking into account that the neutron interacts with each nucleon of the final nucleus of nucleon number A_2

$$V_{f\mu}^{St} = \frac{(2\pi)^3}{V^{3/2}} A_2 F_2(\mathbf{k}_n)\,\delta(\mathbf{k}_n - \mathbf{k}_2)\,. \tag{53}$$

C. Transition probability per unit time of electron assisted neutron exchange process

Substituting the obtained forms of $V_{\mu i}^{Cb}$ and $V_{f\mu}^{St}$ (formulae (48) and (53)) into (11) and using the correspondence $\sum_\mu \to \frac{V}{(2\pi)^3} d^3k_n$ and the $\delta(\mathbf{k}_n - \mathbf{k}_2)$ in the integration over the components of $\mathbf{k}_n$ one gets

$$\begin{aligned} T_{fi} \;=\; & -\frac{4\pi e^2 Z A_2 \widetilde{F}_1(\mathbf{k}_2)\, F_2(\mathbf{k}_2) \sqrt{\frac{F_e(E_{ie})}{F_e(E_{f1})}}}{|\mathbf{k}_{fe} - \mathbf{k}_{ie}|^2 + \lambda^2} \times \\ & \times \frac{(2\pi)^6}{V^3} \frac{\delta(\mathbf{k}_1 + \mathbf{k}_2 + \mathbf{k}_{fe} - \mathbf{k}_{ie})}{(\Delta E_{\mu i})_{\mathbf{k}_n = \mathbf{k}_2}}, \end{aligned} \tag{54}$$

where

$$\widetilde{F}_1(\mathbf{k}_2) = \int \Phi_{i1}(\mathbf{r}_{n1})\, e^{-i\left(\mathbf{k}_2 - \frac{\mathbf{k}_1 + \mathbf{k}_{fe} - \mathbf{k}_{ie}}{A_1 - 1}\right)\cdot \mathbf{r}_{n1}} d^3 r_{n1} \tag{55}$$

and $F_2(\mathbf{k}_2)$ is determined by (23). Here Φ_{i1} and Φ_{f2} in (23) are the initial and final bound neutron states. Substituting the above into (10), using the identities $[\delta(\mathbf{k}_1 + \mathbf{k}_2 + \mathbf{k}_{fe} - \mathbf{k}_{ie})]^2 = \delta(\mathbf{k}_1 + \mathbf{k}_2 + \mathbf{k}_{fe} - \mathbf{k}_{ie})\,\delta(\mathbf{0})$ and $(2\pi)^3 \delta(\mathbf{0}) = V$, the $\sum_f \to \sum_{m_2} \int \left[V/(2\pi)^3\right]^3 d^3k_1 d^3k_2 d^3k_{fe}$ correspondence, averaging over the quantum number m_1 and integrating over the components of $\mathbf{k}_{fe}$ (which gives $\mathbf{k}_{fe} = -\mathbf{k}_1 - \mathbf{k}_2 + \mathbf{k}_{ie}$) one obtains

$$\begin{aligned} W_{fi} \;=\; & \int \frac{64\pi^3 \alpha_f^2 \hbar c^2 Z^2 \sum_{l_2=-m_2}^{l_2=m_2} |F_2(\mathbf{k}_2)|^2}{v_c V \left(|\mathbf{k}_1 + \mathbf{k}_2|^2 + \lambda^2\right)^2 (\Delta E_{\mu i})^2_{\mathbf{k}_n = \mathbf{k}_2}} \\ & \times \left\langle |F_1(\mathbf{k}_2)|^2 \right\rangle \frac{F_e(E_{ie})}{F_e(E_{f1})} A_2^2 r_{A_2} \delta(E_f - \Delta) d^3k_1 d^3k_2, \end{aligned} \tag{56}$$

where A_1, A_2 are the initial atomic masses, l_1, m_1 and l_2, m_2 are the orbit and its projection quantum numbers of the neutron in its initial and final state. For $F_1(\mathbf{k}_2)$, $\left\langle |F_1(\mathbf{k}_2)|^2 \right\rangle$ and $F_2(\mathbf{k}_2)$ see (21), (22) and (23). Taking into account the effect of the number of atoms of atomic number A_2 in the solid target the calculation is similar to the calculation of e.g. the coherent neutron scattering [8] and the $|T_{fi}|^2$ must be multiplied by N_L which is the number

of atomic sites in the crystal and by r_{A_2} which is the relative natural abundance of atoms ${}^{A_2}_{Z}X$. We have used $N_L/V = 2/v_c$ with v_c the volume of the elementary cell of the fcc lattice in which there are two lattice sites in the cases of Ni and Pd investigated.

D. Approximations, identities and relations in calculation of cross section

Now we deal with the energy denominator ($\Delta E_{\mu i}$) in (56) and (20) [see $(12)-(19)$]. The shielding parameter λ is determined by the innermost electronic shell of the atom ${}^{A_1}_{Z}X$ and it can be determined as

$$\lambda = \frac{Z}{a_B}, \tag{57}$$

where $a_B = 0.53 \times 10^{-8}\ cm$ is the Bohr-radius. The integrals in (56) and (20) have accountable contributions if

$$|\mathbf{k}_1 + \mathbf{k}_2| \lesssim \lambda \tag{58}$$

and then $E_{fe} \lesssim \hbar^2\lambda^2/(2m_e) = \frac{1}{2}\alpha_f^2 m_e c^2 Z^2$ which can be neglected in $\Delta E_{\mu i}$ and in the energy Dirac-delta. Thus

$$\Delta E_{\mu i} = \frac{\hbar^2\mathbf{k}_1^2}{2m_1} + \frac{\hbar^2\mathbf{k}_2^2}{2m_n} - \Delta_- + \Delta_n \tag{59}$$

and in the Dirac-delta

$$E_f = \frac{\hbar^2\mathbf{k}_1^2}{2m_1} + \frac{\hbar^2\mathbf{k}_2^2}{2m_2}. \tag{60}$$

In this case $\mathbf{k}_1 = -\mathbf{k}_2 + \delta\mathbf{k}$ with $|\delta\mathbf{k}| = \delta k \sim \lambda$. Using

$$k_1 \simeq k_2 \simeq k_0 = \sqrt{2\mu_{12}\Delta}/\hbar \tag{61}$$

(see below) with $\mu_{12}c^2 = A_{12}m_0c^2$, where $A_{12} = (A_1 - 1)(A_2 + 1)/(A_1 + A_2)$ is the reduced nucleon number, one can conclude that the $\mathbf{k}_2 = -\mathbf{k}_1$ relation fails with a very small error in the cases of events which fulfill condition (58) since $k_1/k_0 \simeq 1$, $k_2/k_0 \simeq 1$, $\delta k/k_0 \sim \lambda/k_0$ and $\lambda/k_0 = \alpha_f Z m_e c^2/\sqrt{2\mu_{12}c^2\Delta} \ll 1$. Consequently, the quantity E_f in the argument of the energy Dirac-delta can be written approximately as

$$E_f = \left(\frac{\hbar^2}{2m_1} + \frac{\hbar^2}{2m_2}\right)\mathbf{k}_2^2 = \frac{\hbar^2c^2\mathbf{k}_2^2}{2A_{12}m_0c^2}. \tag{62}$$

Furthermore taking $A_1/(A_1+1) \simeq 1$

$$\Delta E_{\mu i} = \frac{\hbar^2c^2\mathbf{k}_2^2}{2m_0c^2} - \Delta_- + \Delta_n. \tag{63}$$

We introduce the $\mathbf{Q} = \hbar c \mathbf{k}_2/\Delta$, $\mathbf{P} = \hbar c \left(\delta \mathbf{k}\right)/\Delta$, $\varepsilon_f = E_f/\Delta = \left[\mathbf{Q}^2/\left(2A_{12}m_0c^2\right)\right]\Delta$ and $L = \hbar c\lambda/\Delta$ dimensionless quantities. The energy Dirac-delta modifies as $\delta(E_f - \Delta) = \delta\left[\varepsilon_f\left(\mathbf{Q}\right) - 1\right]/\Delta$. The relation (57) yields $L = \hbar cZ/\left(a_B\Delta\right) = Z\alpha_f m_e c^2/\Delta$ and $Z\alpha_f m_e c^2/\Delta \lesssim 1$. Now we change $d^3k_1 d^3k_2$ to $\left(\frac{\Delta}{\hbar c}\right)^6 d^3Q d^3P$ in the integration in (20), use the $\delta\left[g\left(Q\right)\right] = \delta\left(Q - Q_0\right)/g'\left(Q_0\right)$ identity, where Q_0 is the root of the equation $g\left(Q\right) = 0$ ($k_0 = Q_0\Delta/\left(\hbar c\right)$, see (61)), estimate the integral with respect to the components of $\mathbf{P}$ by

$$\int_0^\infty \frac{4\pi P^2 dP}{\left(P^2 + L^2\right)^2} = \frac{\pi^2}{L} \tag{64}$$

and apply $v_c = d^3/4$ (the volume of unit cell of fcc lattice for Ni and Pd of lattice parameter d).

E. $\left\langle \left|F_1\left(\mathbf{k}_0\right)\right|^2 \right\rangle_{Sh}$ and $\sum_{l_2=-m_2}^{l_2=m_2} \left|F_2\left(\mathbf{k}_0\right)\right|_{Sh}^2$ in single particle shell-model and without LWA

Now we calculate the quantities $\left\langle \left|F_1\left(\mathbf{k}_0\right)\right|^2 \right\rangle_{Sh}$ and $\sum_{l_2=-m_2}^{l_2=m_2} \left|F_2\left(\mathbf{k}_0\right)\right|_{Sh}^2$ in the single particle shell model with isotropic harmonic oscillator potential and without the long wavelength approximation (see definitions: (21), (22) and (23)). Taking into account the spin-orbit coupling in the level scheme the emerging neutron states are $0l$ and $1l$ shell model states in the cases of Ni and Pd to be discussed numerically [9]. So the initial and final neutron states $\left(\Phi_{i1}, \Phi_{f2}\right)$ have the form

$$\Phi_{Sh}\left(\mathbf{r}_{nj}\right) = \frac{R_{n_j l_j}}{r_{nj}} Y_{l_j m_j}\left(\Omega_j\right) \tag{65}$$

where $n_j = 0, 1$ in the cases of $0l$ and $1l$ investigated, respectively, and

$$R_{0l_j} = b_j^{-1/2}\left(\frac{2}{\Gamma(l_j + 3/2)}\right)^{1/2} \varrho_j^{l_j+1} \exp\left(-\frac{1}{2}\varrho_j^2\right), \tag{66}$$

$$\begin{aligned} R_{1l_j} \;=\; & b_j^{-1/2}\left(\frac{2l_j + 3}{\Gamma(l_j + 3/2)}\right)^{1/2} \varrho_j^{l_j+1} \times \\ & \times \left(1 - \frac{2}{2l_j + 3}\varrho_j^2\right) \exp\left(-\frac{1}{2}\varrho_j^2\right) \end{aligned} \tag{67}$$

with $\varrho_j = r_{nj}/b_j$ where $b_j = \sqrt{\hbar/\left(m_0\omega_j\right)}$ [9]. Here ω_j is the angular frequency of the oscillator that is determined by $\hbar\omega_1 = 40A_1^{-1/3}\ MeV$ and $\hbar\omega_2 = 40\left(A_2 + 1\right)^{-1/3}\ MeV$ [10].

(The subscript Sh refers to the shell model.) With the aid of these wave functions and for $n_1 = 0, 1$

$$\left\langle \left| F_1 \left(\mathbf{k}_0 \right) \right|^2 \right\rangle_{Sh} = b_1^3 \frac{2^{l_1+2}}{\sqrt{\pi}\left(2l_1+1\right)!!} 4\pi I_{1,n_1}^2 \tag{68}$$

with

$$I_{1,0} = \int_0^\infty \varrho^{l_1+2} j_{l_1}(k_0 b_1 \frac{A_1}{A_1-1}\varrho) e^{-\frac{1}{2}\varrho^2} d\varrho \tag{69}$$

and

$$\begin{aligned} I_{1,1} \; = \; & \left(l_1 + \frac{3}{2} \right) \int_0^\infty \varrho^{l_1+2} \left(1 - \frac{2}{2l_1+3}\varrho^2 \right) \times \\ & \times j_{l_1}(k_0 b_1 \frac{A_1}{A_1-1}\varrho) e^{-\frac{1}{2}\varrho^2} d\varrho. \end{aligned} \tag{70}$$

Here $j_{l_1}(x) = \sqrt{\frac{\pi}{2x}} J_{l_1+1/2}(x)$ denotes spherical Bessel function with $J_{l_1+1/2}(x)$ the Bessel function of first kind.

Similarly

$$\begin{aligned} \sum_{l_2=-m_2}^{l_2=m_2} \left| F_2 \left(\mathbf{k}_0 \right) \right|_{Sh}^2 \; = \; & b_2 f^2 \frac{2^{l_2+2}\left(2l_2+1\right)}{\sqrt{\pi}\left(2l_2+1\right)!!} \times \\ & \times 4\pi \left(\frac{A_2}{A_2+1} \right)^2 I_{2,n_2}^2 \end{aligned} \tag{71}$$

with

$$I_{2,0} = \int_0^\infty \varrho^{l_2+1} j_{l_2}(k_0 b_2 \varrho) e^{-\frac{1}{2}\varrho^2 - \frac{A_2+1}{A_2}\frac{b_2}{r_0}\varrho} d\varrho \tag{72}$$

and

$$\begin{aligned} I_{2,1} \; = \; & \left(l_2 + \frac{3}{2} \right) \int_0^\infty \varrho^{l_2+1} \left(1 - \frac{2}{2l_2+3}\varrho^2 \right) \times \\ & \times j_{l_2}(k_0 b_2 \varrho) e^{-\frac{1}{2}\varrho^2 - \frac{A_2+1}{A_2}\frac{b_2}{r_0}\varrho} d\varrho. \end{aligned} \tag{73}$$

Substituting the results of (68), (71) and (24) into (26) one gets

$$\begin{aligned} \eta_{l_1,n_1,l_2,n_2}\left(A_1, A_2\right) \; = \; & \frac{2^{l_1+l_2+4}}{\pi\left(2l_1+1\right)!!\left(2l_2+1\right)!!} \times \\ & \times \frac{b_1^3 b_2}{r_0^4} \left(\frac{A_2}{A_2+1} \right)^2 I_{1,n_1}^2 I_{2,n_2}^2. \end{aligned} \tag{74}$$

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