Model for Development of Electric Breakdown in Liquids and Stability Analysis

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MODEL FOR DEVELOPMENT OF ELECTRIC BREAKDOWN IN LIQUIDS AND STABILITY ANALYSIS

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Abstract: A semi-numerical model is presented for the development of electric breakdown in liquids based on nanosecond time scale. In this model, breakdown starts in the pre-existed bubble at the tip of the electrode. Formation of plasma immediately makes the surface of the bubble charged to the potential of the electrode, and the imbalance between the electrostatic force and surface tension makes the bubble elongate at high speed. Numerical estimations made using this model are in good agreement with published experimental data. Linear analysis also shows that the instability of the Rayleigh-Taylor type develops in the plasma-gaseous channel surface points with high curvature.

Keywords: Electric breakdown initiation, nanosecond, electrostatic force, surface tension, stability analysis, capillary wave

1. Introduction

There is increasing interest in the study of electric breakdown in liquids as these processes find more application in industry and academic research. For example, oil filled gaps are used for the insulation of high-voltage devices because of filling oil's higher permittivity [1]; some liquid noble gases are used in nuclear and high-energy physics for radiation detection [2]. More recently, electric breakdown is developed as a physical, non-chemical means of biofoul removal and contaminant reduction in water, with the potential for extension into a wide range of other water treatment applications [3]. In all these applications, it is important to get a better understanding of the key physical mechanisms that take place in the process.

It is generally agreed that the electrical breakdown of liquid involves the generation and growth of a bubble, or more precisely, a cavity at the tip of the electrode in a short time period [4–6]. A number of models for breakdown have been suggested based on microsecond time scale. Some propose that electron avalanche is formed in the liquid phase at the initial stage, and the bubble is generated at the cathode by local heating due to the intense electron emission or perhaps by ion current [4,6]. Others models suggest a bubble dynamic process in which an

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electron avalanche develops in the vapor phase following the formation of a pre-existing bubble [5].

With the advances in image recording technique, recent studies using fast Schlieren-photography shows that the essential of electrical breakdown in liquid involves fast initiation and propagation of low-density filaments (bubbles, cavities, cracks). The breakdown starts within nanoseconds after application of high voltage, and the growth rate of the filament is up to several kilometers per second [7]. A physical intuition and some elementary estimations say that during such a short time, heating is probably not the best explanation of channel formation at the initial stage of a discharge.

The objective of this paper is to present a model for initiation and development of breakdown in liquids subjected to high voltage based on nanosecond time scale. The model is consists of two components: explanation and numerical estimations for propagation of filaments during breakdown and a stability analysis of the filaments.

2. Model

According to the bubble nucleation theory in boiling, micro-bubbles appear and disappear at the nucleation sites at the liquid-solid interface. The number of the bubbles and location depend upon the surface roughness, fluid properties and operation conditions. Surface tension that exists on the interface of a bubble crates additional pressure inside the bubble:

$$P_{in} - P_{\infty} = 2\gamma / r$$

where P_{in} is the pressure inside the bubble, P_{∞} is the ambient pressure, γ is surface tension coefficient of the liquid and r is the radius of the bubble. For water at room temperature γ = 0.078 N/m. To maintain a bubble with radius of 1 micron, the additional pressure inside would be 1.78 atm.

When high voltage is applied to the electrode, breakdown first happens in the gas phase bubbles. The direct ionization rate coefficient k_I of air in the reduced electric field E/n_0 of 10^3 V·cm^2 is in the order of 10^{-10} to 10^{-9} cm^3/s. The molecule density n_0 inside the bubble under previously estimated pressure is in the order of 10^{19} cm^{-3}. The time need for breakdown would be $(k_I\, n_0)^{-1}$, which is in the order of 0.1 to 1 ns. This process is fast enough to explain the initiation developed within nanoseconds observed in [7]. Electrons, due to their high mobility, will move faster and deposit on the gas-liquid interface. As a result, the interface will be charged negatively to the electrode potential. For a typical breakdown voltage Φ_0 = 30 kV and electrode radius r = 1 mm, the electric field at the electrode tip can be estimated as $\Phi_0/r = 3\times10^7$ V/cm. This strong field will push the bubble to form the bush-like structure.

To quantify the process described above, we now define the equations for the formation and propagation of the plasma-filled filaments. Gravity is neglected here because it is very small in comparison with electric forces (see estimations below). Because of external forcing by the electric field, the isolated system of the forming plasma channel does not conserve energy or momentum. First it is assumed that the filament is an elongated object with a rounded tip. When plasma is produced inside the bubble, the gas-liquid interface can be regarded as equipotential with the electrode due to high plasma conductivity.

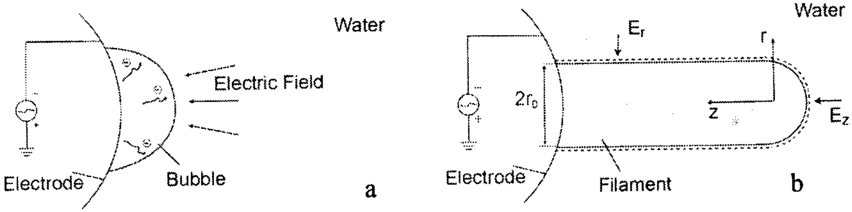

Figure 1. (a) – initial bubble form at the moment of high voltage application; (b) – bubble elongation and gaseous plasma filament formation due to interaction of electrical forces with surface tension and external pressure forces

Assume the charge density both inside and outside the filament surface can be ignored comparing with that on the filament surface. For a slender filament, applying Laplace Equation in the radial direction:

$$\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial\Phi}{\partial r}\right)=0 \tag{1}$$

with boundary condition: $\Phi\big|_{r=r_0}=\Phi_0$ and $\Phi\big|_{r=R}=0$. Φ_0 is the potential at the interface, r_0 is the radius of the filament, and R is the radius where the potential can be regarded as zero.

By solving the equation, the radial electric field E_r and local surface charge density σ can be written as:

$$E_r=\frac{\partial\Phi}{\partial r}=-\frac{\Phi_0}{r_0\ln(R/r_0)} \tag{2}$$

$$\sigma=\varepsilon E_{r_0}=-\varepsilon\frac{\Phi_0}{r_0\ln(R/r_0)} \tag{3}$$

where ε is permittivity of the liquid. Similarly, we can get the electric field and local charge density in the axial direction near the tip of the filament:

$$E_z=\frac{\partial\Phi}{\partial r}=-\frac{\Phi_0}{r_0} \tag{4}$$

$$\sigma=\varepsilon E_{z_0}=-\varepsilon\frac{\Phi_0}{r_0} \tag{5}$$

Since $R \gg r_0$, it is obvious that the electrostatic pressure in the axial direction is much higher than that at the radial direction, and both of them are inversely proportional to r_0^2. At the initial stage of the filament growth r_0 is usually small, which means the bubble will grow in all directions. At some point the electrostatic force will first reach balance with surface tension and ambient pressure in the radial direction, thus keeping constant radius, while the bubble continues growing in the axial direction. Considering the force balance in the axial direction:

$$P + E_r\sigma - \gamma / r_0 = P_\infty \tag{6}$$

where P is the liquid vapor pressure inside the filament (growth of the bubble due to electric force reduces the gas pressure and vapor pressure becomes larger that the gas pressure) , γ is the surface tension of the liquid, and P_∞ is the ambient pressure. Since the liquid vapor pressure is usually small compared to P_∞, the force balance equation is reduced to

$$E_r\sigma - \gamma / r_0 = P_\infty \tag{7}$$

At the tip of the filament, the pressure caused by the axially directed electric force is

$$E_z\sigma = [\ln(R/r_0)]^2 E_r\sigma = [\ln(R/r_0)]^2 (\gamma / r_0 + P_\infty) \tag{8}$$

Although it is convenient to consider the filament to be a tube with constant radius, a more realistic state for experiments thins due to the stress at the interface from the interaction between the hydrodynamic pressure on the elongating bubble and the electric force (Fig. 2). In such a 'needle' shape filament, the hydrodynamics pressure against the driving force of fast propagation is proportional to the tangent of the angle of attack. Assume the hydrodynamics pressure and surface tension is in balance with the electric pressure when the filament growth reaches maximum:

$$\frac{1}{2}\rho v_0^2 \tan\alpha = E_z\sigma_z = \varepsilon\frac{\Phi^2}{r_0^2} \tag{9}$$

where v_0 is the maximum velocity, α is the angle of attack of the growing filament, which can be estimated as $\alpha \approx r_0/L$ (L is the length of the filament). Then the maximum velocity can be written as:

$$v_0 = \frac{\Phi}{r_0}\sqrt{\frac{2\varepsilon_{water}}{\rho \tan\alpha}} \approx \frac{\Phi}{r_0}\sqrt{\frac{2\varepsilon_{water} L}{\rho r_0}} \tag{10}$$

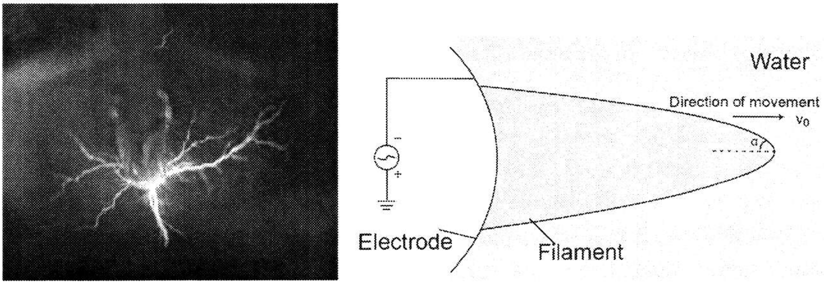

Figure 2. (a) Photo of corona discharge in water; (b) schematic diagram of needle shape filament

At the initiation state of a typical breakdown of water, the radius of the filament is in the order of 1 μ and the length is in the order of 1 mm. As shown in Fig. 3, the relation between growth rate and applied voltage can be calculated, which is in good agreement with experiment results [7] (about 3 km/s at 12 kV).

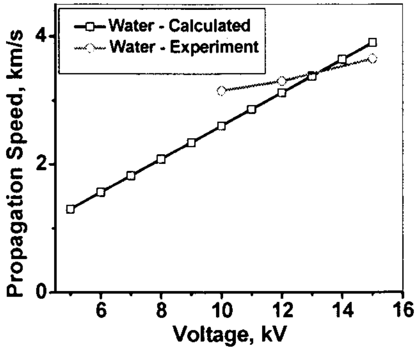

Figure 3. Comparison of calculated and measured propagation speed of filament during breakdown of water

3. Stability Analysis

Now we discuss the linear stability analysis of axisymmetric perturbation of a filament surface that is based on the method used in the papers [8]. Assume a small wave-like disturbance occurred at the surface of cylindrical bubble with initial radius r_0, as shown in Fig. 4. The peak-to-peak amplitude and wave number

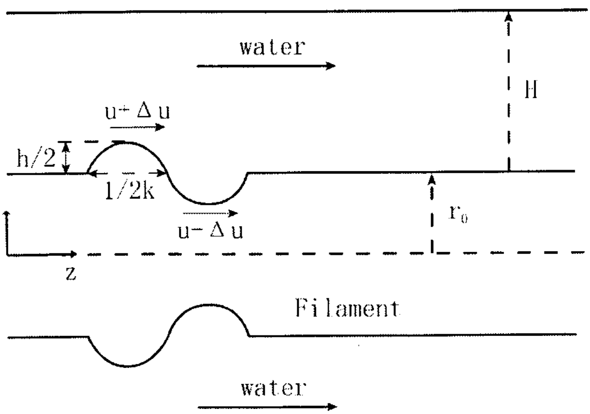

Figure 4. Schematic diagram of disturbance at the surface of filament

of the disturbance is h and k, respectively. H is the depth of wave influence [9], u is the velocity of liquid relative to the disturbance. Then surface of the perturbation is represented by

$$r = r_0 + \frac{h}{2}\exp(ikz + iwt) \tag{11}$$

With the perturbation, the local electrostatic force, surface tension and hydrodynamic pressure will be changed. Generally, the surface tension tends to minimize the surface area and subsequently stabilize the disturbance, while the electrostatic force tends to push the disturbance to grow. In the reference frame that moves together with the disturbance, the effects of these three forces are considered separately, and summed up when it comes to pressure balance between the crest and trough along the stream line.

3.1. ELECTROSTATIC PRESSURE

According to the Equations (2) and (3) from part II, the electrostatic pressure is proportional to the square of local curvature of the interface, which is different at the crest and trough of the perturbation:

$$p_{E,c} = \varepsilon\phi_0^2 \frac{M_c^2}{4} \tag{12}$$

$$p_{E,t} = \varepsilon\phi_0^2 \frac{M_t^2}{4} \tag{13}$$

where $p_{E,c}$ and $p_{E,t}$ is the electrostatic pressure at crest and trough respectively, M_c and M_t is the mean curvature at the crest and trough respectively. The mathematical expression for the mean curvature can be written as [10]:

$$M = \left(\frac{1}{r\sqrt{1+(\partial_z r)^2}} - \frac{\partial_z(\partial_z r)}{(1+(\partial_z r)^2)^{1.5}}\right) = \frac{1}{r} - \partial_z\partial_z r \tag{14}$$

Plug Equation (11) into Equation (14), we can get expressions for M_c and M_t:

$$M_c = \frac{1}{r_0 + \frac{h}{2}} + \frac{h}{2}k^2 \tag{15}$$

$$M_t = \frac{1}{r_0 - \frac{h}{2}} - \frac{h}{2}k^2 \tag{16}$$

Subsequently, $p_{E,c}$ and $p_{E,t}$ can be written as:

$$p_{E,c} = \varepsilon\phi_0^2 \frac{M_c^2}{4} = \varepsilon\phi_0^2 \frac{1}{4}\left(\frac{1}{(r_0+\frac{h}{2})^2} + \frac{hk^2}{r_0+\frac{h}{2}}\right) \quad (17)$$

$$p_{E,t} = \varepsilon\phi_0^2 \frac{M_t^2}{4} = \varepsilon\phi_0^2 \frac{1}{4}\left(\frac{1}{(r_0-\frac{h}{2})^2} - \frac{hk^2}{r_0-\frac{h}{2}}\right) \quad (18)$$

So the electrostatic pressure difference between the crest and trough is:

$$\Delta p_E = -\frac{\varepsilon\phi_0^2 h}{2r_0^3} + \frac{\varepsilon\phi_0^2 hk^2}{2r_0} \quad (19)$$

3.2. STATIC PRESSURE – SURFACE TENSION

Similarly, the surface tension across the interface at the crest and trough can be written as:

$$p_{T,t} = \gamma M_t = \frac{\gamma}{r_0 - \frac{h}{2}} - \frac{h\gamma}{2}k^2 \quad (20)$$

$$p_{T,c} = \gamma M_c = \frac{\gamma}{r_0 + \frac{h}{2}} + \frac{h\gamma}{2}k^2 \quad (21)$$

Then the surface tension difference between the cress and trough is:

$$\Delta p_T = \frac{\gamma h}{r_0^2} - \gamma hk^2 \quad (22)$$

3.3. HYDRODYNAMIC PRESSURE

When there is disturbance on the interface of the filament, the flow speed of liquid will be perturbed in the depth of wave influence, inducing a dynamic pressure difference between the crest and trough:

$$\Delta p_3 = \frac{1}{2}\rho(u + \frac{\Delta u}{2})^2 - \frac{1}{2}\rho(u - \frac{\Delta u}{2})^2 = \rho u \Delta u \quad (23)$$

The dynamic pressure is related to the flow speed through Bernoulli's equation. The pressure difference from the electrostatic force and dynamic effect of the flow has the opposite sign from that due to surface tension. For a balance between two kinds of oppositely directed pressure differences we have:

$$\rho u \Delta u + (-\frac{\varepsilon \phi_0^2 h}{2 r_0^3} + \frac{\varepsilon \phi_0^2 h k^2}{2 r_0} + \frac{\gamma h}{r_0^2} - \gamma h k^2) = 0 \tag{24}$$

Following the steps described in [8] to eliminate Δu, it is easy to get:

$$\rho w^2 = (\gamma - \frac{\varepsilon \phi_0^2}{2 r_0})(k^2 - \frac{1}{r_0^2})k \tag{25}$$

As we described previously, w is the oscillation frequency of the disturbance and it is complex. From Equation (11) it is clear that if the imaginary part of the complex w is not zero, the disturbance will grow exponentially with time. Since the difference between static pressure due to surface tension at the crest is less than that at the trough, it is possible to conclude that $1/k \ll r_0$. The same conclusion can be reached from the small perturbation assumption, and therefore the second factor in the Equation (25) will always be positive. Once the liquid is fixed, the surface tension γ and permittivity ε are constant, so the stability will depend on the applied voltage and radius of the filament. When the voltage exceeds some critical value, w^2 will be negative and the disturbance becomes instable. Another possibility is when the filament radius reduces to certain value, which happens during the propagation, the instability becomes the dominant process and the single filament begins to grow into the bush-like pattern. When the radius goes to infinity, the equation reduces to $\rho w^2 = \gamma k$, which is the formula for classic capillary wave.

Conclusion

The electric breakdown of liquids involves the generation and propagation of vapor-plasma channels through the liquids. The Semi-numerical model described in this paper explains the dynamics of initiation of pulsed electric breakdown in liquids based on nanosecond time scale. Assuming there are pre-existing bubbles at the tip of the electrode, breakdown will first occur inside the bubble in gaseous phase and make the gas-liquid interface equipotential to the electrode. Then the plasma bubble (streamer) will elongate in the axial direction because of imbalance of electrostatic force and surface tension. The estimated streamer velocity is in a good agreement with published experimental results. Linear stability analysis showed that the branching of the filaments can be attributed to the Rayleigh-Taylor type instability, which develops in the plasma-gaseous channel surface points with high curvature.

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