Electromagnetic fields of current structures

N.E. NEVESSKY

Current structures of various complexity are often used as nodes in all kinds of electronic circuits. Each of such structures, powered by alternating current, creates an electromagnetic field in the surrounding space with a characteristic configuration reflecting the features of the emitting structure. Reference books usually provide expressions for the field of a dipole or, at best, for a ring with a current. There are no formulas for fields of current structures of higher order, although these fields are very interesting in their own right.

The interest is due to the fact that, among other reasons, as follows from a number of observations, these fields are biologically active. A probable reason for the activity may be the correspondence between the chiral characteristics of the electromagnetic field and the macromolecules of living matter [1]. Moreover, these fields are interesting because they are determined not only by the first, but also by higher current derivatives. This provides an inductive contribution to the active resistance of the receiving systems, which can be both positive and negative in sign. The latter circumstance can be used to explain the anomalously high energy conversion coefficient observed in electronic systems with resonant transformers [2]. Finally, investigating electromagnetic fields of different current structures, we can experimentally analyze different non-Maxwellian versions of electromagnetic theory, some consequences of which have direct practical interest. These include the charge-equivalent theory of Dokuchaev V.I. [3], "spinor electrodynamics" (Khvorostenko N.P., 1990), "electrodynamics-2" (Nevesky N.E., 1991) and others, not yet published.

In this regard, it seems appropriate to obtain classical expressions for the electric and magnetic fields of current structures of different orders, and the present paper is devoted to this purpose. All calculations are based on Maxwell theory and in a low-frequency approximation: $2\pi c/R_x \gg \omega$, where R_x is the largest is the largest characteristic size of current structure.

1. Dipole. Consider a dipole consisting of a fixed negative charge $(-)q$, located at the origin, and a positive charge $(+)q$ oscillating along the Oz axis (Fig. 1).

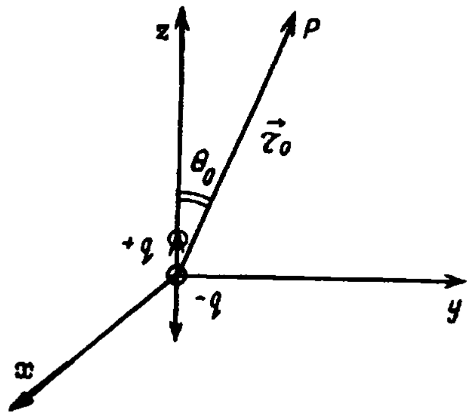

Fig. 1

The potentials φ and A of a point dipole at distances much greater than the maximum displacement of the positive charge from the equilibrium position are obtained directly from the delayed Leenard-Wichart potentials [4].

$$\varphi_d = \left[\frac{D(t')}{r_0^2} + \frac{\dot{D}(t')}{cr_0}\right]\cos\theta_0\,; A_d = \frac{\dot{D}(t')}{cr_0}e_z. \qquad (1)$$

Here D is the dipole moment: $D(t) = q\varepsilon(t),\ \varepsilon(t)$ is positive charge displacement; $t' = t - r_0/c$ is "lagging time".

Accordingly, E and H of the dipole fields, which are equal by definition

$$E = -\nabla\varphi - \frac{1}{c}\frac{\delta A}{\delta t}; \quad H = \mathrm{rot}A, \qquad (2)$$

have the form;

$$\left.\begin{aligned} E_d &= \left[\frac{D(t')}{r_0^3} + \frac{\dot{D}(t')}{cr_0^2}\right]\left(2\cos\theta_0\,\vec{e}_{r_0} + \sin\theta_0\,e_{\theta_0}\right) + \\ &\quad + \frac{\ddot{D}(t')}{c^2 r_0}\sin\theta_0\,e_{\theta_0}; \\ H_d &= \left[\frac{\dot{D}(t')}{cr_0^2} + \frac{\ddot{D}(t')}{c^2 r_0}\right]\sin\theta_0\,e_{\varphi_0}. \end{aligned}\right\} \qquad (3)$$

2. Ring. Consider a ring of radius R, through which current $I(t)$ flows. The fields of an alternating current ring can be found as a superposition of the fields of the dipoles that make up the ring. Such a representation is admissible at low frequencies, when

$R/c\tau_x \ll 1$ (τ_x is the characteristic time) and, consequently, all dipoles can be considered to oscillate in phase (Fig. 2). Then

$$\varphi_K = \oint \varphi_d \frac{N}{2\pi R} dl,$$

where N is the total number of dipoles, or since $dl\vec{e}_\varphi = d\vec{l}$; $\cos\theta = \frac{re_\varphi}{r}$, then

$$\varphi_K = \frac{N}{2\pi r} \oint \left[\frac{\dot{D}(t-r/c)}{cr} + \frac{D(t-r/c)}{r^2}\right] \frac{\vec{r}dl}{r} \equiv 0\,.$$

The integral becomes zero like a circular gradient, since $dl = -dr$. This is, however, the way it should be, since the ring is neutral.

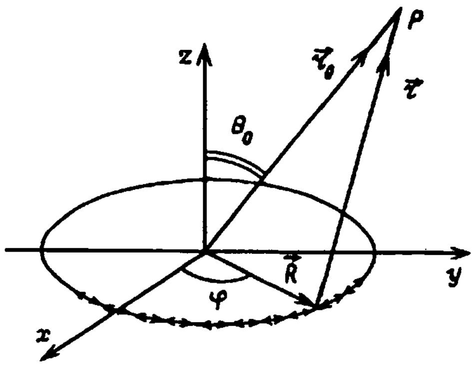

Fig. 2

For A we obtain

$$A_K = \frac{N_K}{2\pi R} \oint A_d dl = \oint \frac{I(t-r/c)}{cr} dl\,, \qquad (5)$$

since, by definition, the current is $I = N\dot{D}/2\pi R$. The value $I(t-r/c)$ cannot simply be taken out of the integral sign. However, it can be expanded in powers of a small parameter $(r_0 - r)/c\tau_x \sim R/c\tau_x \ll 1$:

$$I(t-r/c) = I(t-r_0/c) + \dot{I}(t-r_0/c)\,(r_0-r)/c + \cdots.$$

Substituting this expansion into (4) and taking the integral, we obtain

$$A_K = \left[\frac{I(t')}{r_0^2} + \frac{\dot{I}(t')}{cr_0}\right] \frac{\pi R^2}{c} \sin\theta_0\, e_{\theta\varphi_0}. \qquad (6)$$

Accordingly, E and H fields of the ring are equal:

$$\left.\begin{aligned} E &= -\frac{\pi R^2}{c}\left[\frac{\dot{I}(t')}{cr_0^2} + \frac{\ddot{I}(t')}{c^2 r_0}\right] \sin\theta_0\, e_{\varphi_0}; \\ H &= \frac{\pi R^2}{c}\left\{\left[\frac{I(t')}{r_0^3} + \frac{\dot{I}(t')}{cr_0^2}\right]\left(2\cos\theta_0\, e_{r_0} + \right.\right. \\ &\left.\left. + \sin\theta_0\, e_{\theta_0}\right) + \frac{\ddot{I}(t')}{c^2 r}\sin\theta_0\, e_{\theta_0}\right\}. \end{aligned}\right\} \qquad (7)$$

(Here t', as before, is the lagged time: $t' = t - r_0/c$.)

Comparison of expressions (3) and (7) shows their symmetry. In these expressions, E and H seem to have changed places. The fields for the ring can be obtained from the dipole fields by a formal substitution: $E \to H$; $H \to -E$ and $D \to M_K$, where M_K is the magnetic moment of the ring;

$$M_K = \frac{\pi R^2}{c} I(t') \qquad (8)$$

The magnetic moment of the ring is related to the dipole moment D by the relationship

$$M_K = \frac{NR}{2c}\dot{D}\,.$$

It is interesting that the ring fields depend not only on the first derivative of current $\dot{I}$, but also on the second derivative of $\ddot{I}$. In the far zone the role of $\ddot{I}$ becomes decisive.

3. Torus. Consider a torus of radius R_T composed of N_T rings of radius R and fed by alternating current $I(t)$ (Fig. 3).

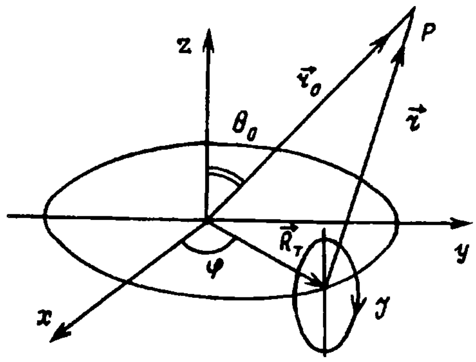

Fig. 3

The fields of the torus can be found as a superposition of the fields of its constituent turns. At low frequencies, when $R < R_T \ll c\tau_x$, we may again assume that the currents in all turns change in phase. In this case

$$\varphi_T = 0; \quad A_T = \oint \frac{N_T}{2\pi} A_K d\varphi\,.$$

Since according to (6) $A_K \sim \sin\theta_0\, e_{\varphi_0} = [e_{z_0} + r_0]/, /r_0$ then, replacing in accordance with Fig. 3 $r_0 \to r;\ e_{z_0} \to e_\varphi$, we get

$$A_T = -\frac{N_T}{2\pi}\oint d\varphi \left[\frac{I(t-r/c)}{r^2} + \frac{\dot I(t-r/c)}{cr}\right]\frac{\pi R^2}{c}\frac{[r\times e_\varphi]}{r}\,.$$

By expanding $I(t-r/c)$ by powers of the small parameter $(r_0 \to r)/c\tau_x$ and taking the integral, we obtain

$$A_T = \frac{N_T R_T}{2c}\pi R^2\left\{\left[\frac{I(t')}{r_0^3} + \frac{\dot I(t')}{cr_0^2}\right](2\cos\theta_0\, e_{r_0} + \sin\theta_0\, e_{\theta_0}) + + \frac{\ddot I(t')}{c^2 r_0}\sin\theta_0\, e_{\theta_0}\right\} \qquad (10)$$

Accordingly, E and H of the torus field are equal:

$$\left.\begin{aligned} E_T &= -\frac{N_T T_T}{2c}\frac{\pi R^2}{c}\left[\frac{\dot I(t')}{r_0^3} + \frac{\ddot I(t')}{cr_0^2}\right](2\cos\theta_0\, e_{r_0} + \\ &\quad + \sin\theta_0\, e_{r_0}) + \frac{\dddot I(t')}{c^2 r_0}\sin\theta_0\, e_{\theta_0}\Big\}; \\ H_T &= -\frac{N_T R_T}{2c}\frac{\pi R^2}{c}\left[\frac{\ddot I(t')}{cr_0^2} + \frac{\dddot I(t')}{c^2 r_0}\right]\sin\theta_0\, e_{\varphi_0}\,. \end{aligned}\right\} \qquad (11)$$

It is remarkable that the fields of the torus are identical in configuration to the fields of an electric dipole with a dipole moment:

$$D_T = \frac{N_T R_T}{2c}\frac{\pi R^2}{c}\dot I(t')\,. \qquad (12)$$

Expressions (11) can be formally obtained from expressions (7) for the ring fields by replacing here

$$E \to H;\ \ H \to (-)E;\ \ M_K \to D_T = \frac{N_T R_T}{2c}\dot M_K\,. \qquad (13)$$

The same procedure, as noted above, relates the fields of the ring to the fields of the dipole.

It is further curious that the expression for the fields of the torus already contains the third derivative of the current $\dddot I$. In the far zone, it becomes decisive. In the near zone, the magnetic field is determined by the second derivative $\ddot I$ (and does not depend on the first derivative at all). The circumstance is important. So, for example, the inductive coupling of two misaligned tori should be determined precisely by the second derivative of the current.

4. Supertor. This is the next most complex current structure. Let us consider a supertor of radius R_s, made of N_s tori (Fig. 4).

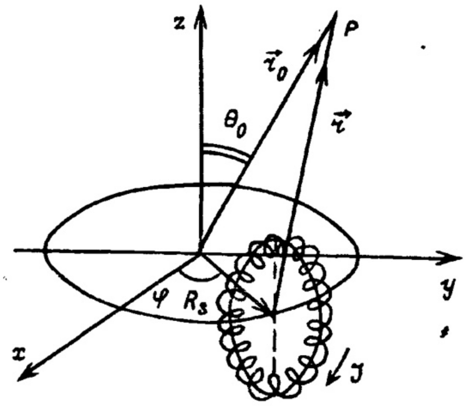

Fig. 4

The procedure for obtaining fields is still the same. We consider, as before, the frequencies are sufficiently low, i.e., $R_s \ll c\tau_x$, and the current in all elements of the supertorus is the same (there is no phase shift). Then

$$A_s = \oint \frac{N_s}{2\pi} A_T d\varphi\,.$$

Substituting here A_T from (10) and substituting here according to Fig. 4 $r_0 \to r;\ e_{z_0} \to e_\varphi$, and hence, $\cos\theta_0 = (re_\varphi)/r$, we obtain

$$A_s = \frac{N_s}{2\pi}\oint d\varphi \frac{N_T R_T}{2}\frac{\pi R^2}{c}\left\{\left[\frac{I(t-r/c)}{r^3} + \frac{\dot I(t-r/c)}{cr^2}\right]\times \times\left(3\frac{(re_\varphi)}{r}e_r - e_\varphi\right) + \frac{\ddot I(t-r/c)}{c^2 r}\left(\frac{re_\varphi}{r}e_r - e_\varphi\right)\right\}.$$

The integral is calculated in the same way as before. Finally, we find

$$A_s = -\frac{N_s R_s}{2}\frac{N_T R_T}{2c}\frac{\pi R^2}{c}\left[\frac{\ddot I(t')}{cr_0^2} + \frac{\dddot I(t')}{c^2 r_0}\right]\sin\theta_0\, e_{\varphi_0}\,. \qquad (14)$$

Accordingly, E and H of the supertor field are equal:

$$\left.\begin{aligned} E_s &= \frac{N_s R_s}{2c}\frac{N_T R_T}{2c}\frac{\pi R^2}{c}\left[\frac{\dddot I(t')}{cr_0^2} + \frac{\ddddot I(t')}{c^2 r_0}\right]\sin\theta_0\, e_{\varphi_0}; \\ H_s &= (-)\frac{N_s R_s}{2c}\frac{N_T R_T}{2c}\frac{\pi R^2}{c}\left\{\left[\frac{\ddot I(t')}{r_0^3} + \frac{\dddot I(t')}{cr_0^2}\right](2\cos\theta_0\, e_{r_0} + \right. \\ &\quad \left. + \sin\theta_0\, e_{\theta_0}) + \frac{\ddddot I(t')}{c^2 r_0}\sin\theta_0\, e_{\theta_0}\right\}. \end{aligned}\right\} \qquad (15)$$

Again, we see the same symmetry. The supertor fields can be obtained from the fields of the previous structure (torus) with the same procedure:

$$E \to H,\ \ H \to (-)E,\ \ D_{\mathrm{T}} \to M_s = \frac{N_s R_s}{2c}\dot{D}_{\mathrm{T}}\,. \quad (16)$$

The fields of the supertor are equivalent to the fields of the ring with magnetic momentum:

$$M_s = \frac{N_s R_s}{2c}\frac{N_{\mathrm{T}} R_{\mathrm{T}}}{2c}\frac{\pi R^2}{c}\ddot{I}(t')\,. \quad (17)$$

The role of higher derivatives of the current has increased even more. The supertorus fields do not depend on the first derivative at all. In the near zone, they are determined by the derivatives $\ddot{I}$ and $\dddot{I}$, and in the far zone, by the fourth derivative of the current $\ddddot{I}$.

The study can be extended further; consider the "supersupertor", i.e., a torus composed of supertori, and so on. However, this is not necessary since the nature of the regularity has already emerged. There is a deep symmetry between the electromagnetic fields of elementary current structures of different orders. If we determine the order of the structure according to the rule: the dipole is the structure of the first order; the ring is a second-order structure; the torus is a third-order structure, and so on, then the following generalization can be made.

The fields of all elementary structures are dipole fields: electric for odd-order structures (dipole, supersupertor, etc.) or magnetic for even-order structures (ring, supertor, etc.).

The fields of the next most complex elementary structure can be obtained from the fields of the previous structure using the recurrent procedure:

$$E_n \to H_{n+1},\ \ H_n \to (-)E_{n+1},\ \ D_{\mathrm{n+1}} = \frac{N_{n+1}R_{n+1}}{2c}\dot{D}_n\,.$$

Physically, the moment D_n is electric for odd n and magnetic for even n. Expressions (3) are valid for a first-order structure, a dipole.

When passing to the structure of the next order, the next derivative of the current also appears every time. In the far zone, the fields of the n-th structure are determined entirely by the n-th derivative of the current: $I^{(n)}$ The fields in the near zone are determined, respectively, by the derivatives and $I^{(n-1)}$ and $I^{(n-2)}$ (only three derivatives always appear in the expressions for the fields: n-th, (n-1)-th and (n-2)-th).

The dependence of the fields of elementary structures on the higher derivatives is significant. The presence of this dependence changes the nature of the inductive coupling of the structures: the odd-order derivatives give an additional contribution to the reactive resistance, while the even-order derivatives $\ddot{I}, \ddddot{I}$ give a contribution to the active resistance. This contribution can be negative (e.g., the contribution from $\ddot{I}$) and therefore reduce the overall resistance of the circuit.

With respect to radiation (far-field), the role of higher derivatives becomes decisive. The Poynting vector in the far field has the form

$$S_n = \frac{1}{4\pi}\frac{\langle(\ddot{D}_n)^2\rangle}{c^3 r_0^2}\sin^2\theta_0\, e_{r_0}\,.$$

Accordingly, its value is proportional to $(\ddot{I})^2$ for a ring, $(\dddot{I})^2$ for a torus, $(\ddddot{I})^2$ for a supertorus, and so on.

Thus, systems composed of elementary current structures should have unusual properties both with respect to their own functioning and with respect to radiation, and this makes them an interesting subject for theoretical and experimental research.

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Author: Nevessky Nikolai Evgenievich graduated faculty of theory and experimental Physics of the Moscow Engineering Physics Institute in. 1972 In 1987 he defended his Ph.D. dissertation on the topic "The influence of acoustic fields for aerosol coagulation" in Odessa state university. Works as a senior Research Fellow in the Department of Theoretical problems of the Russian Academy of Sciences.