Dynamical pair production at sub-barrier energies for light nuclei

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In the collision of two heavy ions the strong repulsion coming from the Coulomb field is enough to produce e^+e^- pair(s) from vacuum fluctuations. The energy is provided by the kinetic energy of the ions and the Coulomb interaction at the production point. If, for instance the electron is located at the center of mass (C.M.) of the two ions moving along the z-axis, and the positron at a distance x from the electron (fig.1), the ions can be accelerated towards each other since the Coulomb barrier is lowered by the presence of the electron. This screening may result in an increase of the fusion probability of light ions above the adiabatic limit.

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The exchange of a virtual photon is responsible for the Coulomb force. In this process virtual electron-positron pairs can be created and annihilated. These virtual charges polarize the vacuum, resulting in a correction to the $1/r$ potential. Uehling was the first to derive the vacuum polarization correction to first order in the fine structure constant α [1] important in the analysis of $p-p$ scattering data [2]. In ref. [3] we showed that the vacuum polarization correction is on the order of one percent of the Coulomb energy in nuclear collision systems. This value seems small, but the strong fields in fission processes can be on the order of 200 MeV. A correction on the order of 2 MeV could noticeably affect the height of the Coulomb barrier, where the nuclear and Coulomb energies roughly cancel. A lower/higher Coulomb barrier increases/decreases the cross-section of sub-barrier fusion, e.g. Carbon-Carbon fusion in the cores of stars.

The vacuum polarization is not just a perturbative effect; production of real e^+e^- pairs can occur during the dynamics in the presence of strong fields, when the available energy exceeds twice the electron mass [4–7]. In the 1980s, experimentalists at GSI found some anomalous production of e^+e^- pairs in heavy ion collisions. Various explanations were proposed, including production of a hypothesized new light particle and experimental error [8]. To our knowledge, there is no consensus [9–12]. In this first paper we discuss the non-perturbative calculation of pair production for light nuclei. We will show that in opportune conditions, the pair may screen the Coulomb repulsion between the ions giving them an extra acceleration towards each other. This effect may increase the fusion cross section above the adiabatic limit [13–18].

For the positron to become a real particle, it must tunnel from the vacuum through the Coulomb barrier and leave the electron behind. We will compute the probability of tunneling through this barrier. We have two nuclei each with charge $+Ze$ (for simplicity) a distance R apart (fig. 1). We assume that the electron is at the center of mass of the two nuclei, and the positron is tunnelling on a line perpendicular to the beam axis. The distance from the electron to the positron is labeled by the coordinate x. The Coulomb energy of the positron is (in units where $4\pi\varepsilon_0 = 1$)

$$V_+(R,x) = \frac{2Ze^2}{\sqrt{\left(\frac{R}{2}\right)^2 + x^2}} - S(x)\frac{e^2}{x}, \tag{1}$$

where $S(x)$ is a screening factor to be discussed in the sequel. When the positron emerges from the barrier, it can have a momentum p_T perpendicular to the x-axis, and the electron will have momentum $-p_T$. To a good approximation [4, 5] the positron satisfies the Klein Gordon (K.G.) equation with energy E_+,

$$\left[(E_+ - V_+(R,x))^2 - p_x^2 - m_T^2\right]\psi = 0, \tag{2}$$

where $m_T = \sqrt{m^2 + p_T^2}$ is the transverse mass of the positron. The Dirac equation leads to the K.G. equation with an extra term $\boldsymbol{\alpha}\cdot\nabla V$, which comes from the spinor nature of the fermion wave function and gives only high-order effects in the tunneling probability thus it is neglected in this paper [5]. Following Wong [5], we divide Eq. (2) by $-2m_T$ to obtain

$$\left[\frac{p_x^2}{2m_T} + \frac{m_T}{2} - \frac{(E_+ - V_+(R,x))^2}{2m_T}\right]\psi = 0. \tag{3}$$

We have implicitly factored out the transverse plane wave part of the wave function. This is formally equivalent to the (time independent) Schrödinger equation

$$\left(\frac{p_x^2}{2m_T} + V_{eff}\right)\psi = E_{eff}\psi, \tag{4}$$

for a particle of mass m_T in an effective potential of

$$V_{eff}(x) = \frac{m_T}{2} - \frac{(E_+ - V_+(R,x))^2}{2m_T}, \tag{5}$$

with energy $E_{eff} = 0$. The classical turning points are where $V_{eff} = 0$, or

$$E_+ - V_+(R,x) = \pm m_T. \tag{6}$$

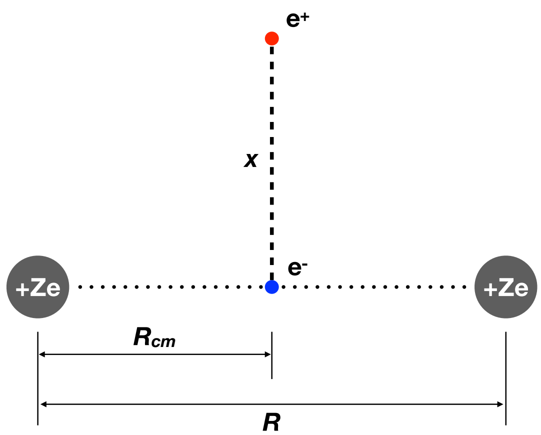

FIG. 1: (Color online) Geometry of the pair production.

The maximum occurs when $E_+ = V_+(R,x)$ and the barrier height is given by $m_T/2$.

Considering a positron in an occupied negative energy state at $x=0$, by definition, it has energy $E_+ \leq V_+(R,0) - m_T$. In order to tunnel into the positive energy region, its energy must be $E_+ \geq m_T$. Taking these conditions together, we obtain a constraint on V_+ for pair production, namely,

$$V_+(R,0) \geq 2m_T. \tag{7}$$

This result confirms the intuition that pair production is possible for electrostatic energies exceedingly twice the mass of the electron.

The total energy before e^+e^- production is

$$E_{cm} = E_k + V_{II}(R) = \frac{P^2}{2\mu} + \frac{Z^2e^2}{R}. \tag{8}$$

Where μ is the reduced mass of the colliding ions and we assume $R \geq R_1 + R_2$, the nuclear radii, i.e. beam energies below the Coulomb barrier. The e^+e^- are produced with transverse mass m_T at a relative distance x_e which will be discussed below. The potential energy seen by the positron is

$$V_+(R,x_e) = \frac{2Ze^2}{\sqrt{\left(\frac{R}{2}\right)^2 + x_e^2}} - S(x_e)\frac{e^2}{x_e}. \tag{9}$$

The energy of the positron is the sum of its mass, kinetic energy, and potential energy,

$$E_+ = m_T + V_+(R,x_e). \tag{10}$$

We have introduced a dynamical screening factor $S(x) = 1 - \exp(-x/x_s)$. The choice $x_s = \frac{e^2}{2m_T}$, sometimes called the classical screening value, implies that for $x \to 0$, the e^+e^- are on top of each other and the mass is given by the Coulomb screened interaction, $S(x)e^2/x \to 2m_T$ which is the energy needed from an external source (the Coulomb field of the ions) to produce the pair, see eq. (7). This assumption ensures energy conservation avoiding the ultraviolet divergence of the Coulomb field. This screening could come from the virtual particles in the vacuum. For instance, we imagine the vacuum as containing a density of pairs proportional to the energy density of the Coulomb field, which in our units is

$$u = \frac{1}{8\pi}\mathcal{E}^2. \tag{11}$$

A typical value for the electric field in our system is

$$\mathcal{E} = \frac{Ze}{(R/2)^2}. \tag{12}$$

We divide the energy density u by $2m_T$ to get the number density of e^+e^- pairs

$$n_p = \frac{Z^2e^2}{\pi R^4 m_T} = \frac{2Z^2x_s}{\pi R^4}. \tag{13}$$

The pairs we consider in our model originate in the region between the two nuclei, which we model as a cylinder of radius x_s and length R. Multiplying the volume of this cylinder by the density of pairs obtained previously, we obtain the expected number of virtual pairs available to tunnel

$$N_p = \frac{2Z^2x_s^3}{R^3}. \tag{14}$$

For two uranium nuclei with their surfaces touching, $N_p \approx 14$ [9], for carbon in the same configuration, $N_p \approx 1.2$. This low value for light ions barely justifies a perturbative treatment of the production and we are going to use it to normalize the predicted cross section for pair production.

Since the total energy must be conserved, after production we have:

$$E_{cm} = E'_k + V_{II}(R) + V_+(R,x_e) - \frac{4Ze^2}{R} + 2m_T. \tag{15}$$

The system is completely symmetric, but a small fluctuation will push the e^+ away from the e^- due to the Coulomb repulsion between the positron and the ions. The positron tunnels through the Coulomb barrier and exits at x_e where its (and the electron's) momentum along the x-direction is zero, fig. 1. At x_e, the total energy is given by eq. (15). If the positron is very fast compared to the ion motion, then we can assume the ions do not move much.

A microscopic calculation is needed to determine the final energy distribution between the electron and the positron. Our approximation is good if m_T is large so that the pair has a good amount of kinetic energy when it is created. Notice that in the case of very large m_T, the e^+ and e^- emerge at about 180^o in the center of mass frame. Comparing our various expressions for the energy, eqs (8), (15), we find the kinetic energy of the ions changes by an amount

$$E'_k - E_k = -\left(V_+(R, x_e) - \frac{4Ze^2}{R} + 2m_T\right). \tag{16}$$

Since $V_+(R, x_e) = E_+ - m_T$, we can also rewrite this as

$$\Delta E_k = E'_k - E_k = -\left(E_+ + m_T - \frac{4Ze^2}{R}\right). \tag{17}$$

And

$$E_+ = \frac{4Ze^2}{R} - m_T - \Delta E_k \geq m_T. \tag{18}$$

The last condition gives

$$R \leq \frac{4Ze^2}{2m_T + \Delta E_k}, \tag{19}$$

that is the largest distance for which the production may occur. We stress that other pair's configurations are of course possible, for instance exchanging the positron and the electron in fig. 1. Different configurations cost more energy and are less probable but calculations can be easily performed for any configuration.

For illustration, we enforce the condition $V_{eff} = 0$ at $x = 0$. There can be two solutions corresponding to

$$E_+ = \frac{4Ze^2}{R} - 2m_T \pm m_T. \tag{20}$$

Thus, according to eqs. (17, 18), the ions either *gain* $2m_T$ of kinetic energy, or there is no change in kinetic energy at the moment of production. This situation is very interesting especially in the sub-barrier fusion of light nuclei since, even in the case of zero kinetic energy gain from the ions, the presence of the electron in the middle of the two ions lowers the Coulomb barrier thus enhancing the fusion probability [13, 18]. We are interested in unbound positrons with $E_+ > m_T$. This requirement together with eq. (20) gives a maximum transverse mass for dynamical pair production

$$m_{T,max} = \frac{2Ze^2}{R}. \tag{21}$$

Since our model only includes the Coulomb force between the ions, we only consider $R > R_1 + R_2$ where the nuclear force is not as important. For two ^{12}C nuclei with their surfaces touching, eq. (21) gives a maximum transverse mass of 3.14 MeV. For ^{238}U in the same condition, $m_{T,max}$ = 17.8 MeV. The corresponding effective potential for the two solutions is

$$V^{(1)}_{eff}(R, x) = \frac{m_T}{2} - \frac{[m_T + V_+(R, x) - \frac{4Ze^2}{R}]^2}{2m_T}, \tag{22}$$

$$V^{(2)}_{eff}(R, x) = \frac{m_T}{2} - \frac{[3m_T + V_+(R, x) - \frac{4Ze^2}{R}]^2}{2m_T}. \tag{23}$$

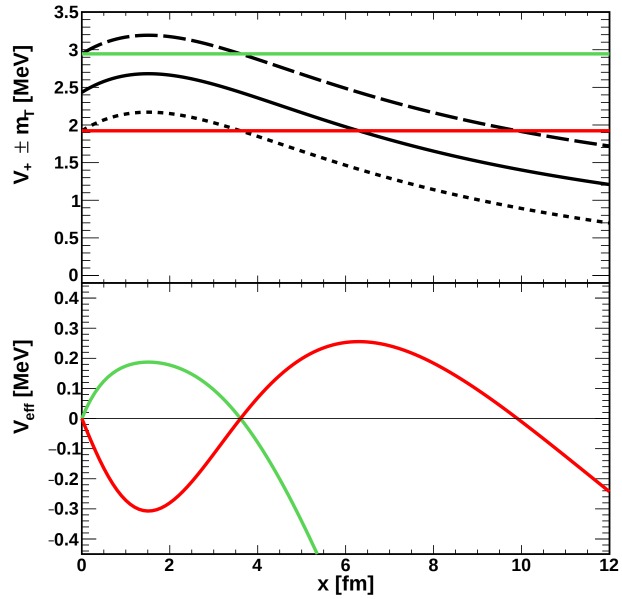

FIG. 2: (Color online) An illustrative example when $V_{eff}(R, 0) = 0$. In the bottom panel we plot V_{eff} vs x and the corresponding potential with full line ($\pm m_T = m_e$-top panel, dashed and dotted lines) seen by the positron. The calculations are performed for ^{12}C+^{12}C collisions.

In figure 2, we plot the effective potential (bottom panel) and the potential ($\pm m_T$-top panel) vs the relative distance between the pair for the case discussed above. The only acceptable solution is the lowest one given by the red line. A simple inspection of the top panel shows that the positron for this case is initially in the negative energy region and tunnels to the positive one. The other solution gives the positron already in the positive energy region (green line) thus is not allowed by our proposed mechanism. Other possible solutions can be found if $V_{eff}(R, x = 0) < 0$. From this discussion we learned that the two ions can *gain* kinetic energy because of the location of the electron (in the middle) and the positron (away from the ions), fig. 1, and may enhance the sub-barrier fusion probability.

The tunneling probability for the positron is given by:

$$\Pi_t = [1 + \exp(2A)]^{-1}, \tag{24}$$

where A is the imaginary action integrated between the turning points of the effective potential, see for instance fig. 2-bottom. The action can be calculated numerically, and some case results are displayed in fig. 3 with $m_T = m_e$. In the calculations different values of $\Delta E_k = E'_k - E_k$, see eq. (18), have been assumed. The lowest value of R is given by the classical turning point, i.e.:

$$R_i = \frac{Z^2e^2}{E_{c.m.}}. \tag{25}$$

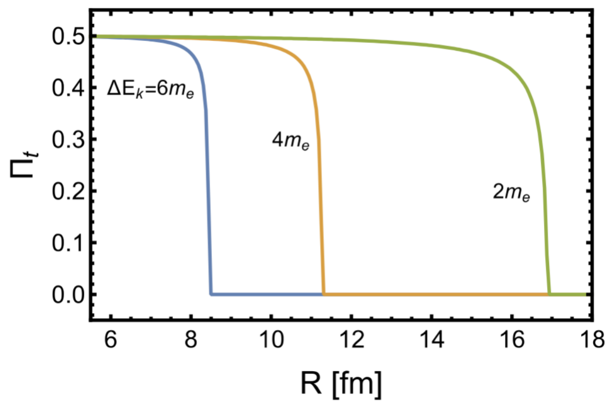

FIG. 3: (Color online) Tunnelling probability for the positron as function of the relative distance of the two C ions, and $E_{c.m.}$=9.4 MeV and different values of ΔE_k.

Since $E_+ \geq m_T$, we can easily estimate the value of R where the probability becomes zero:

$$R_x = \frac{4Ze^2}{\Delta E_k + 2m_T}, \tag{26}$$

which is consistent with eq. (19). It is easy to show that $R_x \geq R_i$ if:

$$E_{c.m.} \geq \frac{Z}{4}(\Delta E_k + 2m_T). \tag{27}$$

From these results and figure 3 we can safely assume that $\Pi_t = 0.5$ for $R_i < R < R_x$.

It should not be surprising that the probability is of the order of 0.5 since the maximum height of the barrier is $m_T/2$, see eq. (5), and the barrier width of the order of 10 fm. The probability goes to zero when $E_+ \to m_T$ and $x \to \infty$. This also agrees with our estimate of the number of pairs produced in the cylinder of radius x_s which for C+C is of the order of one. For heavier nuclei this calculation must be performed for each distance and all the created pairs must be followed microscopically since barriers may be modified by the presence of previously created pairs and there may be not enough energy to produce another pair after the first one. The probability cutoffs in the figure are essentially determined by energy conservation for each value of ΔE_k.

Since this is a dynamical process, times are important. A characteristic time for pair production is given by the Heisenberg principle:

$$\Delta\tau = \frac{\hbar}{2m_T}, \tag{28}$$

thus, the rate at which a given virtual pair can attempt to tunnel is $\Delta\tau^{-1}$.

There is a second characteristic time for the tunnelling process. A simple inspection of fig. 2, bottom, shows that the positron may be trapped by the Coulomb barrier up to the inner turning point. This is analogous to the number of assaults per unit time in the theory of alpha decay, fission etc. This quantity may be estimated by the ratio of the distance travelled by the positron before hitting the inner barrier (of the order of few Fermis from figure 2) divided by its average speed. For transverse masses equal to the rest mass of the electron, the corresponding time is smaller than the time obtained from the Heisenberg uncertainty principle, and we will use the value given in eq. (28) for an estimate of the cross section. Microscopic dynamical calculations are needed for heavier systems when more than one pair may be produced and energy conservation must be fulfilled.

Here we will use simple and transparent physical arguments to estimate the value of the cross section for pair production. We write the cross section as:

$$\sigma(E_{c.m.}) = \frac{\pi\hbar^2}{2\mu E_{c.m.}} \sum_{l=0}^{n} (2l+1)\Pi_l P_H. \tag{29}$$

Since we are interested in sub-barrier reactions, we consider the $l = 0$ case only and we fix $\Pi_0 = \Pi_t = 0.5$ as discussed above, and $P_H = \tau/\Delta\tau$, see eq. (28). Thus, in order to estimate the cross section we need the average τ it takes to the ions to travel from R_x to R_i, eqs. (25, 26). The total distance travelled is $\Delta R = 2(R_x - R_i)$, the factor of two is because the pair can be produced during the approaching or rebounding phase of the two ions. Similarly, we can estimate the velocity as the average at the closest point $v(R_i)(=0)$ or at R_x, $v(R_x) = \sqrt{2/\mu(E_{c.m.} - V_{II}(R_x))}$, thus $\overline{v} = \frac{v(R_i)+2v(R_x)}{3}$ and $\tau = \frac{1}{N_{\pm}^{max}} \frac{\Delta R}{\overline{v}}$. $N_{\pm}^{max}$ is a parameter determined requiring that the total probability, i.e. the largest number of produced pair possible without violating energy conservation: $N_{\pm}^0 = \Pi_0 P_H \leq 1$. With these approximations and $m_T = m_e$, the cross section is:

$$\begin{aligned} \sigma_0(E_{c.m.}) &= \frac{\pi\hbar^2}{2\mu E_{c.m.}} 0.5 \frac{\tau}{\Delta\tau} \\ &= \frac{1}{N_{\pm}^{max}} \frac{6\pi\hbar Z e^2 m_e}{\sqrt{2\mu} E_{c.m.}^2 (\Delta E_k + 2m_e)} \\ &\times \sqrt{E_{c.m.} - \frac{Z}{4}(\Delta E_k + 2m_e)}. \end{aligned} \tag{30}$$

We stress that this approximation is good for $Z \geq 4$. For smaller nuclei we need to replace R_x with $R'_x = \frac{Z^2 e^2}{2m_T + \Delta E_k}$ as dictated by energy conservation in the production process. For $^{12}C+^{12}C$, we find the maximum number of pairs produced in the collisions by summing over the trajectory without taking into account the energy loss after a pair is produced. From the estimate, eq. (16) we found that at most 1 pair is created in $^{12}C+^{12}C$ which is obtained near $E_{c.m.}$=2 MeV (ΔE_k=0 MeV) in fig. 4. Thus we normalize the number of maximum pairs produced at this energy to one. Clearly the maximum

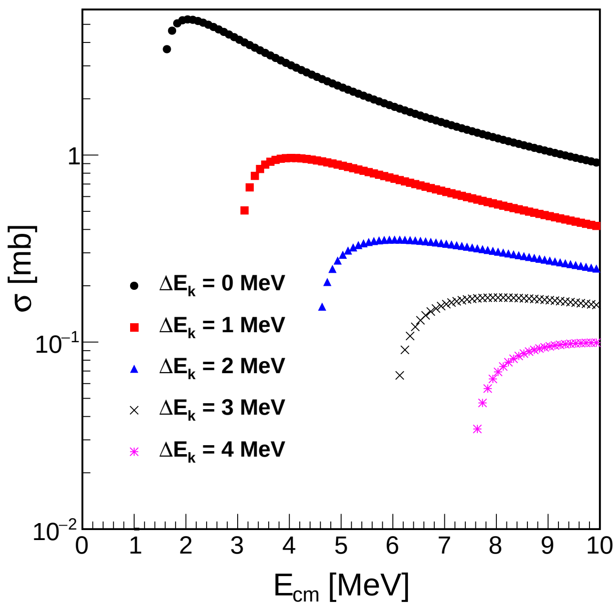

FIG. 4: (Color online) Upper limit for the integrated cross section for e^+e^- production in ^{12}C+^{12}C scattering below the Coulomb barrier for different values of ΔE_k.

number of pairs produced in the collisions and the relative cross section of fig. 4 critically depends on the ultraviolet cutoff x_s discussed above and it must be confirmed or modified by future experimental data. Furthermore, microscopic calculations able to follow the heavy ion trajectory and the dynamics of one or more pairs created during the time evolution must be implemented in order to make prediction also for heavier colliding nuclei and collisions of different mass number nuclei.

In conclusion, we have discussed pair production from vacuum within the Schwinger formalism. We have shown the conditions for tunneling and the possibility that if the electron is situated at the c.m. of the colliding nuclei, extra screening may occur. This screening may enhance sub-barrier fusion of light nuclei above the adiabatic limit. For ^{12}C+^{12}C collisions we predict $E_{c.m.} \geq 1$ MeV for this effect to occur. The cross sections are of the order of mb or less. These predictions call for detailed experimental investigation of pair production for this system and their energies also in coincidence with fusion fragments to be able to extract correlation functions. An enhancement may be revealed by performing a correlation between fusion events with and without pair production.

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