

# Nuclear processes in solids: basic 2nd-order processes

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## Abstract

Nuclear processes in solid environment are investigated. It is shown that if a slow, quasi-free heavy particle of positive charge interacts with a "free" electron of a metallic host, it can obtain such a great magnitude of momentum in its intermediate state that the probability of its nuclear reaction with an other positively charged, slow, heavy particle can significantly increase. It is also shown that if a quasi-free heavy particle of positive charge of intermediately low energy interacts with a heavy particle of positive charge of the solid host, it can obtain much greater momentum relative to the former case in the intermediate state and consequently, the probability of a nuclear reaction with a positively charged, heavy particle can even more increase. This mechanism opens the door to a great variety of nuclear processes which up till now are thought to have negligible rate at low energies. Low energy nuclear reactions allowed by the Coulomb assistance of heavy charged particles is partly overviewed. Nuclear  $pd$  and  $dd$  reactions are investigated numerically. It was found that the leading channel in all the discussed charged particle assisted  $dd$  reactions is the electron assisted  $d + d \rightarrow {}^4He$  process.

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## I. INTRODUCTION

It is a standard of nuclear physics that heavy, charged particles  $j$  and  $k$  of like positive charge of charge numbers  $z_j$  and  $z_k$  need considerable amount of relative kinetic energy  $E$  determined by the height of the Coulomb barrier in order to let the probability of a nuclear interaction have significant value. Mathematically it appears in the energy dependence of the cross section ( $\sigma$ ) of the charged-particle induced reactions as

$$\sigma(E) = S(E) \exp[-2\pi\eta_{jk}(E)]/E, \quad (1)$$

where  $S(E)$  is the astrophysical factor, which can be written as  $S(E) = S(0) + S_1E + S_2E^2$ , [1]. The Sommerfeld parameter

$$\eta_{jk}(k) = z_j z_k \alpha_f \frac{\mu_{jk} c}{\hbar k}, \quad (2)$$

where  $k = |\mathbf{k}_j - \mathbf{k}_k|$  is the magnitude of the relative wave vector  $\mathbf{k} = \mathbf{k}_j - \mathbf{k}_k$  of the interacting particles of wave vectors  $\mathbf{k}_j$  and  $\mathbf{k}_k$ , ( $k \sim \sqrt{E}$ ). The reduced mass of particles  $j$  and  $k$  rest masses  $m_j$  and  $m_k$

$$\mu_{jk} = m_j m_k / (m_j + m_k), \quad (3)$$

$\hbar$  is the reduced Planck constant,  $e$  is the elementary charge and  $\alpha_f$  is the fine structure constant. (It can be shown that in the case of slow relative motion the exponential function in (1) is the same as the Gamow factor [2], that hinders nuclear reactions between particles of like electric charge.) The energy dependence of the cross section (1) can be derived applying the Coulomb solution

$$\varphi(\mathbf{r}) = e^{i\mathbf{k}\cdot\mathbf{r}} f(\mathbf{k}, \mathbf{r}) / \sqrt{V}, \quad (4)$$

which is the wave function of a free particle of charge number  $z_j$  in a repulsive Coulomb field of charge number  $z_k$ , where  $V$  denotes the volume of normalization, and  $\mathbf{r}$  is the relative coordinate of the two particles. Here

$$f(\mathbf{k}, \mathbf{r}) = e^{-\pi\eta_{jk}/2} \Gamma(1 + i\eta_{jk}) {}_1F_1(-i\eta_{jk}, 1; i[kr - \mathbf{k}\cdot\mathbf{r}]), \quad (5)$$

where  ${}_1F_1$  is the confluent hypergeometric function and  $\Gamma$  is the Gamma function [3].

It is the consequence of energy dependence (1) of the cross section that to this day it is a commonplace that the rate of any nuclear reaction between heavy, charged particles of positive charge is unobservable at low energies. The aim of this paper is to show that

in a solid (particularly in a metal), contrary to the former assumption, there are nuclear processes that can have observable rate at low energies.

## II. PRELIMINARY CONSIDERATIONS

In the low-energy range ( $kR \ll 1$ , where  $R$  is the radius of a nucleon) and for  $|\mathbf{r}| \leq R$  the long wavelength approximation

$$|\varphi(\mathbf{r})| = |\varphi(\mathbf{0})| = f_{jk}(k)/\sqrt{V} \quad (6)$$

is valid, where

$$f_{jk}(k) = |f(\mathbf{k}, \mathbf{0})| = \sqrt{\frac{2\pi\eta_{jk}(k)}{\exp[2\pi\eta_{jk}(k)] - 1}} \quad (7)$$

is the Coulomb factor. We introduce the notation

$$F_{jk}(k) = f_{jk}^2(k) \quad (8)$$

with which the cross section (and the rate) of a first order process is proportional. If  $k$  is small then the long wavelength approximation produces the form (1) of  $\sigma$  well. Thus the fact that the rate of any nuclear reaction between heavy, charged particles of positive charge is unobservable at low energies is the consequence of  $F_{jk}(k)$  being small.

In solids, however, where free electrons are present nuclei also Coulomb interact with electrons. It means that the Hamiltonian governing the state of the nuclei must also contain the interaction Hamiltonian with the electrons. If one wants to describe the interaction (scattering) of a nucleus with an other one that takes into consideration the interaction with the electrons the lower order process appears in the perturbation calculation can be seen in FIG. 1. The basic idea can be demonstrated with the aid of FIG.1(a), in which a Coulomb scattering is followed by a capture process governed by strong interaction. When calculating the transition probability (and the rate) of such a second order process the following statements are valid. Energy and momentum (wave number vector) are conserved, i.e.  $E_i = E_f$ ,  $\mathbf{k}_i = \mathbf{k}_f$ , where  $E_i$  and  $E_f$  are the total energies, and  $\mathbf{k}_i$  and  $\mathbf{k}_f$  are the total wave number vectors in the initial and final states, respectively. However energy-wave number vector (momentum) conservation may be violated in the "intermediate" state. In the cases investigated the initial particles (particles 1, 2 and 3) are slow and the sum of their

initial kinetic energies  $E_i$  and the sum of their wave number vectors  $\mathbf{k}_i$  can be neglected, i.e.  $E_i = 0$  and  $\mathbf{k}_i = 0$  can be supposed.

It is thought that particle 1 is an electron and particles 2 and 3 are heavy, and of positive charge. The nuclear reaction  $2 + 3 \rightarrow 4$  has reaction energy  $\Delta$ . This energy is shared between the outgoing particles 1 and 4. Thus particle 1 obtains energy and wave number vector of nuclear order of magnitude. Since the Coulomb interaction in the case of free particles conserves wave number vector (momentum), and since the initial wave number vector of particles 1 and 2 can be neglected in wave number conservation, particle 2' gets a wave number vector  $\mathbf{k}_{2'}$  opposite to the final wave number vector  $\mathbf{k}_{1'}$  of particle 1', i.e.  $\mathbf{k}_{2'} = -\mathbf{k}_{1'}$ . Moreover if one calculates the Coulomb matrix element using plane waves for the free particles then the matrix element must be corrected with the so called Sommerfeld factor [4]

$$g_S = \frac{f_{12}(|\mathbf{k}_2 - \mathbf{k}_1|)}{f_{12}(|\mathbf{k}_{2'} - \mathbf{k}_{1'}|)}, \quad (9)$$

where  $f_{12}$ -s are Coulomb factors [see (7)] for particles 1 and 2 of electric charge numbers  $z_2 = 1$  and  $z_1 = -1$  since particle 1 is an electron. If particle 2 is heavy and slow, and particle 1 is an electron then  $|\mathbf{k}_2 - \mathbf{k}_1| = k_1$ , that is the magnitude of the initial wave number vector of particle 1, furthermore  $|\mathbf{k}_{2'} - \mathbf{k}_{1'}| = 2k_{2'}$  since  $\mathbf{k}_{2'} = -\mathbf{k}_{1'}$ . Thus the cross section and the rate are proportional to

$$G_S(k_1, 2k_{2'}) = g_S^2 = \frac{F_{12}(k_1)}{F_{12}(2k_{2'})}. \quad (10)$$

It can be shown that  $F_{12}(2k_{2'}) = 1$  and therefore  $G_S = F_{12}(k_1) = 23.18 \times (E_1^{-1/2} (eV))$ , where  $E_1$  is the energy of the initial free electron. When calculating the matrix element of the strong interaction potential between particles 2 and 3 we use (6). Consequently the second order rate is proportional to  $G_S F_{23}$  which is mainly determined by  $F_{23}(k_{2'}) \simeq \exp[-2\pi\eta_{23}(k_{2'})]$ . Now  $k_{2'} = |\mathbf{k}_{1'}|$  has a nuclear order of magnitude.

The first order rate is proportional to  $F_{23}(k) \simeq \exp[-2\pi\eta_{23}(k)]$ , where  $k$  is the wave number of the slow, initial particle 2 (particle 3 is supposed to be at rest). Since  $k \ll k_{2'}$ , and therefore  $\eta_{23}(k) \gg \eta_{23}(k_{2'})$ , and consequently  $\exp[-2\pi\eta_{23}(k)] \ll \exp[-2\pi\eta_{23}(k_{2'})]$ , the rate of the second order process is much higher than that of the first order process. As a result, although the rate of a second order process is usually much less than the rate of a first order process, in this case the exponential increment is so huge that it can dwarf the rate of the first order process.

As a numerical example we consider the electron assisted  $d + d \rightarrow {}^4_2He$  process with slow deuterons. In this case, one of the slow deuterons (as particle 2) can enter into Coulomb interaction with a quasi-free, slow electron of the solid before the nuclear reaction (see FIG. 1(a)). The states of the free deuteron and the free electron can be described by plane waves, therefore the Coulomb interaction preserves the wave number vector (momentum). If in the second order process the Coulomb interaction is followed by strong interaction, which induces a nuclear capture process, then the energy  $\Delta$  of the nuclear reaction is divided between the electron and the heavy nuclear product. Since the rest mass  $m_N$  of the nuclear product is much larger than the rest mass  $m_e$  of the electron, the electron will take almost all the total nuclear reaction energy  $\Delta$  away and the magnitude of its wave number vector  $k_{1'} = \sqrt{\Delta^2 + 2m_e c^2 \Delta} / (\hbar c) \simeq \Delta / (\hbar c)$  (if  $\Delta \gg m_e c^2$ ). If initially (before the Coulomb interaction) the electron and the deuteron move slowly and the magnitudes of their wave number vectors are much smaller than  $\Delta / (\hbar c)$ , then the initial wave number vectors can be neglected in the wave number vector (momentum) conservation and consequently, in the intermediate state the deuteron gets a wave number vector of magnitude  $k_{1'}$  and of direction opposite to the wave number vector of the electron. Thus the deuteron will have a (virtual) wave number vector in the intermediate state that is large enough to make it able to overcome the Coulomb barrier by tunneling and to take part in a nuclear process. If  $\Delta = 23.84$  [MeV], which is the reaction energy of the  $d + d \rightarrow {}^4_2He$  reaction, then the deuteron will have a (virtual) wave number  $k_{2'} = \Delta / (\hbar c)$  in the intermediate state (in state 2'). The corresponding value  $F_{23} = 0.356$ . It must be compared e.g. to the extremely small value  $F_{23}(1 \text{ eV}) = 1.1 \times 10^{-427}$  that is characteristic of the first order process.

### III. ELECTRON ASSISTED NUCLEAR PROCESSES

The change of state of heavy charged particles induced by solid state environment is modelled in the following way. Let us take two independent systems  $A$  and  $B$ , where  $A$  is a solid and  $B$  is an ensemble of free, heavy charged particles (e.g. a free deuteron or proton gas) with the corresponding Hamiltonians  $H_A$  and  $H_B$ . It is supposed that their eigenvalue problems are solved, and the complete set of the eigenvectors of the two independent systems are known. Let us extend the state vectors of systems  $A$  and  $B$  to those nuclear bound states, which are initially empty, corresponding to the assumption that at the beginning the two

systems do not interact. The interaction between them to be switched on adiabatically is described by the interaction Hamiltonian  $V_{AB} = V^{Cb}(\mathbf{x}_{AB}) + V^{St}(\mathbf{x}_{AB})$ , where  $V^{Cb}$  and  $V^{St}$  stand for the Coulomb and the strong interaction potentials, respectively, and the suffixes  $A$  and  $B$  in their argument symbolize that one party of the interaction comes from system  $A$  and the other from system  $B$ . (Similar model is used by [5] introducing the reduced density operator.) In the process investigated, first a heavy, charged particle of system  $B$  takes part in a Coulomb scattering with any charged particle of system  $A$  and it is followed by a strong interaction with some nucleus of system  $A$  that leads to their final bound states. The graphs of the process can be seen in FIG. 1. The process of FIG. 1(a) is a nuclear capture process and the process of FIG. 1(b) is a nuclear reaction. (The processes, where the nuclear interaction is followed by the Coulomb interaction, can be neglected because of the situation discussed in the introduction.) Particles 1 and 3, belonging to system  $A$  are: electrons (e.g. free electrons in the case of a metal) and localized heavy, charged particles (bound, localized  $p$ ,  $d$  and other nuclei) as nuclear targets. Particle 2 belongs to system  $B$ , that is a charged, heavy particle (e.g. proton ( $p$ ) or deuteron ( $d$ )), and that is supposed to move freely in a solid (e.g. in a metal). Since the aim of this paper is to show the fundamentals of the main effect, the problem, that there may be identical, indistinguishable particles in systems  $A$  and  $B$  is not considered, the simplest description is chosen, and the dynamic evolution of the number  $N_2$  of particles 2 of system  $B$  is not investigated.

The transition probability per unit time ( $W_{fi}^{(2)}$ ) of the process can be written as

$$W_{fi}^{(2)} = \frac{2\pi}{\hbar} \sum_f |T_{if}^{(2)}|^2 \delta(E_f - \Delta) \quad (11)$$

with

$$T_{if}^{(2)} = \sum_{\mu} \frac{V_{f\mu}^{St} V_{\mu i}^{Cb}}{E_{\mu} - E_i} \frac{(2\pi)^3}{V} \delta(\mathbf{k}_{1f} + \mathbf{K}_{(type)}), \quad (12)$$

where  $E_i$ ,  $E_{\mu}$  and  $E_f$  are the kinetic energies in the initial, intermediate and final states, respectively,  $\Delta$  is the reaction energy, i.e. the difference between the rest energies of the initial and final states,  $V$  is the volume of normalization.  $V_{\mu i}^{Cb}$  is the matrix element of the Coulomb potential between the initial and intermediate states and  $V_{f\mu}^{St}$  is the matrix element of the potential of the strong interaction between intermediate and final states.  $type = \alpha, +\beta, n\beta$  correspond to the reaction of FIG. 1(a) ( $\alpha$ ) and to the reactions of FIG.

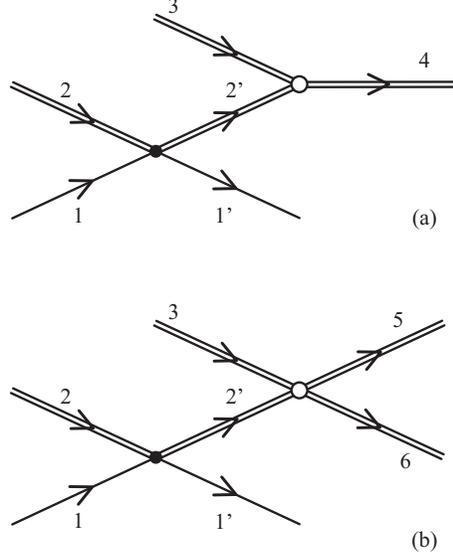


FIG. 1: The graphs of electron assisted nuclear reactions. The simple lines represent free (initial (1) and final (1')) electrons. The doubled lines represent free, heavy, charged initial (2) particles (such as p, d), their intermediate state (2'), target nuclei (3) and reaction products (4, 5, 6). The filled dot denotes Coulomb-interaction and the open circle denotes nuclear (strong) interaction. FIG. 1(a) is a capture process and FIG. 1(b) is a reaction.

1(b) with both particles charged ( $+\beta$ ) and with one of them neutral ( $n\beta$ ), respectively.

$$E_f = E_{1f} + E_{(type)}, \quad (13)$$

$$E_\mu = E_{1f}(\mathbf{k}_{1f}) + E_2(\mathbf{k}_{2\mu}), \quad (14)$$

where  $E_{1f}$  is the kinetic energy and  $\mathbf{k}_{1f}$  is the wave vector of particle 1 (electron) in the final state.  $E_{(\alpha)} = E_4(\mathbf{k}_4)$ ,  $\mathbf{K}_{(\alpha)} = \mathbf{k}_4$  in the case of process of FIG. 1(a) and  $E_{(+\beta \text{ or } n\beta)} = E_5(\mathbf{k}_5) + E_6(\mathbf{k}_6)$ ,  $\mathbf{K}_{(+\beta \text{ or } n\beta)} = \mathbf{k}_5 + \mathbf{k}_6$  in the case of process of FIG. 1(b) with  $E_j(\mathbf{k}_j)$  the kinetic energy of the  $j$ -th particle in the intermediate (particle 2) and final (particles 4 or 5, 6) states. Since particle 1 is an electron,

$$E_{1f} = \sqrt{(\hbar c)^2 k_{1f}^2 + m_e^2 c^4} - m_e c^2 \quad (15)$$

with  $m_e c^2$  denoting the rest energy of the electron.

For the Coulomb potential we use its screened form

$$V^{Cb}(\mathbf{x}) = \int \frac{4\pi e^2 z_1 z_2}{q^2 + \lambda^2} \exp(i\mathbf{q} \cdot \mathbf{x}) d\mathbf{q} \quad (16)$$

with screening parameter  $\lambda$  and coupling strength  $e^2 = \alpha_f \hbar c$ . For the strong interaction the interaction potential

$$V^{St}(\mathbf{x}) = -2f^2 \frac{\exp(-s|\mathbf{x}|)}{|\mathbf{x}|} \quad (17)$$

is applied, where the strong coupling strength  $f^2 = 0.08\hbar c$  [6] and  $1/s$  is the range of the strong interaction. The calculation of the total rates of the electron assisted nuclear processes can be found in Appendix I.

The result of the total rate of the leading, electron assisted ( $p$  or  $d$ ) capture process

$$W_{tot}^{(2)}(\alpha) = K_{tot}(\alpha) \langle G_S(k_{1i}, 2k_{1f}) \rangle_{av} \frac{F_{23}(k_{1f})}{\Delta^4} h_{corr,3}^2 u N_2 \quad (18)$$

with  $k_{1f} = \frac{\Delta}{\hbar c}$ . Here  $u$  denotes the deuteron (or proton) over metal number densities,  $N_2$  is the number of initial particles 2 and for  $G_S$  see (10). Furthermore, we introduced the following notation

$$K_{tot}(\alpha) = \frac{32}{d^6} g_e K_0(\alpha), \quad (19)$$

where  $g_e$  is the number of the valence electron states corresponding to one unit cell and

$$K_0(\alpha) = 192 (2\pi)^8 z_1^2 z_2^2 \left(1 - \frac{2}{e}\right)^2 \alpha_f^2 \left(\frac{f^2}{\hbar c}\right)^2 (\hbar c)^4 R c. \quad (20)$$

Here  $R = 1.2 \times 10^{-13}$  [cm] is the radius of a nucleon (corresponding to the single nucleon approach applied). Finally,  $h_{corr,3} = A_3 - z_3$  in the case of proton capture process and  $h_{corr,3} = A_3$  in the case of deuteron capture reactions (both are taken in the Weisskopf approximation), where  $A_3$  and  $z_3$  are the mass and charge numbers of particle 3.

Taking  $z_2 = 1$ ,  $\alpha_f = 1/137$ ,  $f^2/(\hbar c) = 0.08$ ,  $c = 3 \times 10^{10}$  [cm/s],  $v_c = d^3/4$  (the volume of unit cell for  $Ni$  and  $Pd$ ) with  $d = 3.89 \times 10^{-8}$  [cm] ( $Pd$  lattice) one gets  $K_{tot}(\alpha) = 560$  [ $MeV^4 s^{-1}$ ] in the case of  $Pd$ . Furthermore  $\langle G_S \rangle_{av} = \langle F_{12}(k_{1i}) \rangle_{av} = 23.18 \times \left\langle E_1^{-1/2} (eV) \right\rangle_{av}$ , where  $E_1$  is the energy of the initial free electron and the average is made by means of Fermi-Dirac distribution. ( $N_2$  and  $F_{23}(k_{1f} = \frac{\Delta}{\hbar c})$  are dimensionless and  $\Delta$  has to be substituted in [ $MeV$ ] units in (18).)

The quantity  $F_{23}(k_{1f} = \frac{\Delta}{\hbar c})$  given by (8) is the most important factor in  $W_{tot}^{(2)}(\alpha)$ . The electron (particle 1) transfers large momentum (energy) to particle 2' through Coulomb scattering. Consequently, particle 2' appears in the nuclear process with the corresponding large wave number and the probability of the nuclear process drastically increases.  $F_{23}(\frac{\Delta}{\hbar c})$  has to be compared with  $F_{23}(k_i(E_i))$ , which is the square of the Coulomb factor of the usual

first order process, where  $E_i$  is the kinetic energy in the relative motion of particles 2 and 3 in the initial state of the usual, first order process. If  $E_i$  has  $eV$  order of magnitude then

$$F_{23}\left(\frac{\Delta}{\hbar c}\right) \gg F_{23}(k_i(E_i)). \quad (21)$$

From this relationship it follows that although the usual first order  $2+3 \rightarrow 4$  nuclear process is very strongly hindered by the repulsive Coulomb interaction due to the extremely small value of  $F_{23}(k_i(E_i))$ , in consequence of the appearance of the much larger quantity  $F_{23}\left(\frac{\Delta}{\hbar c}\right)$  in the rate of the second order electron assisted process the hindering effect practically disappears.

#### IV. HEAVY PARTICLE ASSISTED NUCLEAR PROCESSES

In electron assisted nuclear reactions heavy, charged particles are created in the decelerating process of reaction products of the electron assisted processes (e.g.  ${}^4_2He$  of energy of  $0.0758 MeV$  is created in the electron assisted  $d + d \rightarrow {}^4_2He$  reaction). The energy of these heavy particles may be intermediately low (of about  $0.01 [MeV]$ ) so their nuclear processes have to be also considered among the accountable nuclear processes. The corresponding graphs can be seen in FIG. 2. The graph of FIG. 2(a) depicts a nuclear capture process and FIG. 2(b) shows a nuclear reaction. (The processes, where the nuclear interaction is followed by the Coulomb interaction, can also be neglected.) Now particles 1 and 3 belong to system  $A$  and particle 2 which is a free, heavy particle created in an electron assisted nuclear process belongs to system  $B$ . According to the applied notation, particles 2 and 3 take part in a nuclear process and particle 1 only assists it. The different processes will be distinguished by the type of the assisting particle and also by the type of the nuclear process. In our model charged, heavy particles, such as protons ( $p$ ), deuterons ( $d$ ) can form system  $B$ , they may be particle 2, which are supposed to move freely in a solid (e.g. in a metal). The particles, that take part in the processes and belong to system  $A$  are: localized heavy, charged particles (bound, localized  $p$ ,  $d$  and other nuclei) as the participants of Coulomb scattering (particle 1) and localized heavy, charged particles (bound, localized  $p$ ,  $d$  and other nuclei) as nuclear targets (particle 3). The problem, that there may be identical particles in systems  $A$  and  $B$  that are indistinguishable, is also disregarded here.

The calculation of the transition probability per unit time  $\left(W_{fi}^{(2),h}\right)$  of the process can

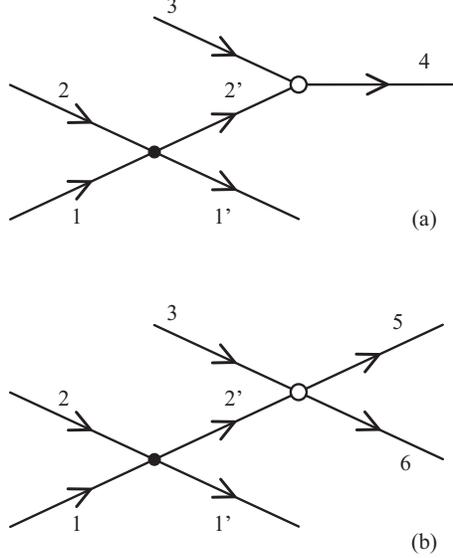


FIG. 2: The graphs of heavy particle Coulomb-assisted nuclear processes. The filled dot denotes Coulomb-interaction and the open circle denotes nuclear (strong) interaction. The localized particles 1 and 3 belong to system A. The free particle 2 (e.g. p or d) belongs to system B. All the particles are heavy and positively charged. From the point of view of the nuclear reaction particle 2' is ingoing, particle 3 is target and particles 4, 5, 6 are products of the process. FIG. 1(a) is a capture process and FIG. 1(b) is a reaction.

be performed through similar steps to those applied for the calculation of the rate of the electron assisted process. The main difference is that particle 1 is heavy and localized.

The matrix element of the screened Coulomb potential modifies as

$$V_{\mu i}^{Cb} = \frac{2(2\pi)^4 e^2 z_1 z_2 \tilde{\psi}_{1i}(\mathbf{k}_{2i} - \mathbf{k}_{1f} - \mathbf{k}_{2\mu})}{V^{3/2} (|\mathbf{k}_{2i} - \mathbf{k}_{2\mu}|^2 + \lambda^2)} g_S. \quad (22)$$

Here

$$\tilde{\psi}_{1i}(\mathbf{K}) = \int \psi_{1i}(\mathbf{x}) e^{-i\mathbf{K}\cdot\mathbf{x}} d\mathbf{x}, \quad (23)$$

where  $\psi_{1i}(\mathbf{x})$  stands for the initial, localized state of particle 1. It is supposed that

$$\psi_{1i}(\mathbf{x}) = \left(\frac{\beta_1^2}{\pi}\right)^{3/4} e^{-\frac{\beta_1^2}{2}\mathbf{x}^2} \quad (24)$$

is the wave function of the ground state of a 3-dimensional harmonic oscillator of angular frequency  $\omega_1$  with  $\beta_1 = \sqrt{m_1\omega_1/\hbar}$  [7]. The calculation of the total rates of the heavy particle assisted nuclear processes can be found in Appendix II.

The total rate of the leading, heavy particle assisted nuclear capture processes valid in the case of *fcc* metals

$$W_{tot}^{(2),h}(\alpha) = \frac{8K_0^h(\alpha)\beta_1^3}{d^3\sqrt{m_0c^2}}G_S(k_{2i}, 2k_{1f})h_{corr,3}^2 \times \left(\frac{a_{12}}{a_{14}}\right)^2 \frac{(z_1z_2)^2}{\sqrt{a_{14}}} \frac{F_{23}^h(\Delta)}{\Delta^{7/2}} uN_2. \quad (25)$$

Here the parameter  $\beta_1$ , e.g. in the case of a localized deuteron, is  $\beta_1^d = \sqrt{m_d\omega/\hbar}$ , where  $m_d$  is the deuteron rest mass,  $\hbar\omega$  corresponds to the energy of an optical phonon in the deuterized metal, and  $d$  is the lattice parameter,

$$K_0^h(\alpha) = K_0(\alpha)/(2\pi)^{3/2}. \quad (26)$$

For  $K_0(\alpha)$  see (20). The quantity

$$F_{23}^h(\Delta) = \frac{2\pi\eta_{23}^h(\Delta)}{\exp[2\pi\eta_{23}^h(\Delta)] - 1} \quad (27)$$

with the parameter

$$\eta_{23}^h(\Delta) = z_2z_3\alpha_f a_{23} \sqrt{\frac{m_0c^2}{2a_{14}\Delta}}, \quad (28)$$

where  $m_0c^2 = 931.494 [MeV]$  is the atomic mass unit, and

$$a_{jk} = \frac{A_j A_k}{A_j + A_k} \quad (29)$$

is the reduced mass number of particles  $j$  and  $k$  of mass numbers  $A_j$  and  $A_k$ .

It can be seen from (27) and (28) that this process opens the door for a great variety of nuclear processes. To get an order of magnitude estimation of the effect we take  $A_1 \simeq A_3 \gg A_2$ , thus  $a_{23} = A_2$  and  $a_{14} = A_4/2 \simeq A_3/2 = z_3$ . With these approximations

$$2\pi\eta_{23,app}^h = 2\pi z_2\alpha_f A_2 \sqrt{\frac{z_3 m_0 c^2}{2\Delta_{app}}}. \quad (30)$$

We take as a typical value  $\Delta_{app} = 4 [MeV]$  in (30) that yields  $2\pi\eta_{23,app}^h = 0.497 \times z_2 A_2 z_3^{1/2}$ . If  $2\pi\eta_{23,app}^h \leq 15$ , one can obtain  $F_{23}^h(\Delta) \geq 4.59 \times 10^{-5}$ , that can be produced with any pair of heavy particles 2 and 3 of  $z_2 A_2 z_3^{1/2} \leq 30$ . From this condition one can draw an important and surprising conclusion: it allows a great variety of nuclear processes, that were thought improbable up till now. Moreover, in many cases  $2\pi\eta_{23}^h \ll 1$ , consequently  $F_{23}^h(\Delta) = 1$  (see

(27)) and the hindering role of  $F_{23}^h(\Delta)$  disappears. In order to show the capability of the heavy particle assisted nuclear processes, some cases of the proton assisted proton captures



are investigated in Appendix III.

## V. CHARGED PARTICLE ASSISTED $dd$ AND $pd$ FUSION PROCESSES IN SOLID METALS

As an example, let us take the simplest charged particle processes, the usual nuclear fusion processes:



The coefficients of the astrophysical factor (see Introduction) of these nuclear fusion processes are available [1]. Owing to their astrophysical importance the low energy range of the above fusion processes has been extensively investigated. In the extremely low energy range, i.e. at near room temperature, the  $S(E) = S(0)$  approximation is valid and the ratio of the rates of the processes (32), (33) and (34) is determined by their  $S(0)$  values and therefore it can be considered energy independent. The corresponding  $S(0)$  values of processes (32), (33), (34) and (35) are:  $5.0 \times 10^{-9}$ ,  $5.6 \times 10^{-2}$ ,  $5.5 \times 10^{-2}$  and  $2.0 \times 10^{-7}$  in  $[MeVb]$  units [1]. Consequently, in a deuteron plasma or in a proton-deuteron mixed plasma the leading processes are the processes (33) and (34) with approximately the same rates, and the rates of processes (32) and (35) are many orders of magnitude smaller.

For comparison we have calculated the  $S(0)$  values of the above processes in the simple nuclear model used in this article and supposing magnetic type transition in reactions (32)

and (35). The results are:  $5.2 \times 10^{-8}$ ,  $5.0 \times 10^{-3}$ ,  $4.4 \times 10^{-3}$  and  $4.5 \times 10^{-9}$  in  $[MeVb]$  units corresponding to reactions (32), (33), (34) and (35), respectively, indicating that our simple model is able to give qualitative or semi quantitative conclusions.

Now we review the ratios of the rates of the processes obtained in solid metals. In the case of the second order, simple electron assisted nuclear processes we have obtained for the relevant quantities (of the corresponding rates, see (48), (68) and (71))  $F_{23}(\frac{\Delta}{\hbar c}) = 0.356$ ,  $K_2 K_+ J_{22} = 1.38 \times 10^{-4}$  and  $K_2 J_{32} = 5.79 \times 10^{-4}$  for the electron assisted versions of processes (32), (33), (34), respectively, and  $F_{23}(\frac{\Delta}{\hbar c}) = 0.0282$  for the electron assisted version of process (35) (see Appendix I). From these numerical values one can conclude that, contrary to the above, in the family of second order electron assisted  $dd$  processes the electron assisted version of process (32) is leading, and the electron assisted versions of processes (33) and (34) have much lower rate. The total rate of the electron assisted version of (32) is

$$W_{tot}^{(2)}(\alpha) = 0.012 \times \left\langle E_1^{-1/2} [eV] \right\rangle_{av} u N_2 [s^{-1}], \quad (36)$$

where  $E_1$  is the energy of the initial free electron in the conduction band. Averaging  $E_1^{-1/2}$  by means of the Fermi-Dirac distribution in the Sommerfeld free electron model at  $T = 0$  yields  $\left\langle E_1^{-1/2} [eV] \right\rangle_{av} = 2 \left( E_F^{-1/2} [eV] \right)$ , where  $E_F$  denotes the Fermi energy [8].

In the case of heavy particle assisted processes we have obtained  $F_{23}^h(\Delta) = 0.902$ , and  $W_{tot}^{(2),h}(\alpha) = 0.0026 \times G_S N_2 u [s^{-1}]$  for process  $d + d \rightarrow {}^4He$  in the case of a deuterized  $Pd$  target, i.e. the particles 1, 2 and 3 are all deuterons. In the case of process  $p + d \rightarrow {}^3He$  and in the same target material, i.e. particles 1 and 3 are deuterons, and particle 2 is a proton  $F_{23}^h(\Delta) = 0.846$  and  $W_{tot}^{(2),h}(\alpha) = 0.378 \times G_S N_2 u [s^{-1}]$ .

From the above rates one can conclude that if energetic, heavy charged particles are present in the sample the heavy particle assisted processes are not negligible and among all the charged particle assisted processes discussed here the electron assisted version of the (32) process is the leading one.

## VI. SUMMARY

It is found that, contrary to the commonly accepted opinion, in a solid metal surrounding nuclear reactions can happen between heavy, charged particles of like (positive) charge of low initial energy. It is recognized, that one of the participant particles of a nuclear reaction

of low initial energy may pick up great momentum in a Coulomb scattering process on a free, third particle of the surroundings. The virtually acquired great momentum, that is determined by the energy of the reaction, can help to overcome the hindering Coulomb barrier and can highly increase the rate of the nuclear reaction even in cases when the rate would be otherwise negligible. It is found that the electron assisted  $d + d \rightarrow {}^4He$  process has the leading rate. In the reactions discussed energetic charged particles are created, that can become (directly or after Coulomb collisions) the source of heavy charged particles of intermediately low (of about a few  $keV$ ) energy. These heavy particles can assist nuclear reactions too. It is worth mentioning that the shielding of the Coulomb potential has no effect on the mechanisms discussed.

Our thoughts were motivated by our former theoretical findings [9] according to which the leading channel of the  $p + d \rightarrow {}^3He$  reaction in solid environment is the so called solid state internal conversion process, an adapted version of ordinary internal conversion process [10]. In the process formerly discussed [9] if the reaction takes place in solid material, in which instead of the emission of a  $\gamma$  photon, the nuclear energy is taken away by an electron of the environment (the metal), the Coulomb interaction induces a  $p + d \rightarrow {}^3He$  nuclear transition. The processes discussed here can be considered as an alternative version of the solid state internal conversion process since it is thought that one party of the initial particles of the nuclear process takes part in Coulomb interaction with a charged particle of the solid material (e.g. of a metal).

There may be many fields of physics where the traces of the proposed mechanism may have been previously appeared. It is not the aim of this work to give a systematic overview these fields. We only mention here two of them that are thought to be partly related or explained by the processes proposed. The first is the so called anomalous screening effect observed in low energy accelerator physics investigating astrophysical factors of nuclear reactions of low atomic numbers [11]. The other one is the family of low energy nuclear fusion processes. The physical background, discussed in the Introduction and in the first part of Section V., was questioned by the two decade old announcement [12] on excess heat generation due to nuclear fusion reaction of deuterons at deuterized Pd cathodes during electrolysis at near room temperature. The paper [12] initiated continuous experimental work whose results were summarized recently [13]. The mechanisms discussed here can explain some of the main problems raised in [13]. (a) The mechanisms proposed here make low energy fusion

reactions and nuclear transmutations possible. (b) The processes discussed explain the lack of the normally expected reaction products.

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## VII. APPENDIX I. - RATE CALCULATION OF ELECTRON ASSISTED NUCLEAR PROCESSES

For particles 1 and 2 (electron and ingoing heavy particle) taking part in Coulomb interaction we use plane waves. Thus the Coulomb matrix element is calculated in the Born approximation

$$V_{\mu i}^{Cb} = \frac{2(2\pi)^7 e^2 z_1 z_2}{V^2} \frac{\delta(\mathbf{k}_{2i} + \mathbf{k}_{1i} - \mathbf{k}_{1f} - \mathbf{k}_{2\mu})}{|\mathbf{k}_{1f} - \mathbf{k}_{1i}|^2 + \lambda^2} g_S, \quad (37)$$

which is corrected with the so called Sommerfeld factor  $g_S$  (see (9)). In the intermediate state we use plane waves of wave vector  $\mathbf{k}_{2\mu}$  for particle 2. The final state of the electron is also a plane wave. The Dirac delta  $\delta(\mathbf{k}_{2i} + \mathbf{k}_{1i} - \mathbf{k}_{1f} - \mathbf{k}_{2\mu})$  will result  $\mathbf{k}_{2\mu} = \mathbf{k}_{2i} + \mathbf{k}_{1i} - \mathbf{k}_{1f}$  in the intermediate state (after integration over  $\mathbf{k}_{2\mu}$ , see later), i.e. in the intermediate state particle 2' may have large momentum, which is determined by the reaction energy due to the final wave vector of the electron.

When calculating the matrix element of the strong interaction potential between particles 2 and 3 we use the approximate form of  $\varphi(\mathbf{r})$  given in (6).

For the process 1(a) the Weisskopf approximation is used, i.e. for the final nuclear state of one nucleon (of particle 4) we take

$$\Phi_{fW}(\mathbf{r}) = \sqrt{\frac{3}{4\pi R^3}} \quad (38)$$

if  $r \leq R$ , where  $R$  is the nucleon radius, and  $\Phi_{fW}(\mathbf{r}) = 0$  for  $r > R$ . In evaluating  $V_{f\mu}^{St}$  the long wavelength approximation ( $\exp(i\mathbf{k}_{2\mu} \cdot \mathbf{x}) = 1$ ) is used with  $sR = 1$  that results

$$V_{f\mu}^{St,W}(\alpha) = -2f^2 f_{23}(k_{2\mu}) \sqrt{\frac{12\pi R}{V}} \left(1 - \frac{2}{e}\right) \quad (39)$$

in single nucleon approach. In the case of process 1(b), for the final states plane waves are assumed producing

$$V_{f\mu}^{St,W}(\beta) = -2f^2 f_{23}(k_{2\mu}) \frac{4\pi R^2}{V} \quad (40)$$

with a neutron as one of the particles 5 and 6. If both particles 5 and 6 have positive charge then the expression of  $V_{f\mu}^{St,W}(\beta)$  must be multiplied by  $f_{56}$ . The wave vectors  $\mathbf{k}_{1i}$ ,  $\mathbf{k}_{2i}$  of the

particles 1 and 2 in their initial state can be neglected in the calculation since  $|\mathbf{k}_{1i}| \ll |\mathbf{k}_{1f}|$ ,  $|\mathbf{k}_{2i}| \ll |\mathbf{k}_{1f}|$ , furthermore  $\lambda \ll |\mathbf{k}_{1f}|$ . Collecting everything obtained above and applying the  $\sum_{\mu} \rightarrow \int [V/(2\pi)^3] d\mathbf{k}_{2\mu}$  correspondence

$$T_{if}^{(2)}(type) = B(type) z_1 z_2 \delta(\mathbf{k}_{1f} + \mathbf{K}_{(type)}) \times \int \frac{g_S f_{23}(k_{2\mu}) \delta(\mathbf{k}_{1f} + \mathbf{k}_{2\mu})}{(E_{\mu} - E_i) |\mathbf{k}_{1f}|^2} d\mathbf{k}_{2\mu} \quad (41)$$

where

$$B(\alpha) = B_0 \frac{\sqrt{3\pi R}}{V^{5/2}} \left(1 - \frac{2}{e}\right), \quad (42)$$

with

$$B_0 = -8 (2\pi)^7 \alpha_f \left(\frac{f^2}{\hbar c}\right) (\hbar c)^2, \\ B(n\beta) = B_0 \frac{2\pi R^2}{V^3}, \quad (43)$$

and

$$B(+\beta) = B(n\beta) f_{56}. \quad (44)$$

For more precise result, beyond the Weisskopf and long wavelength approximations, and beyond the single nucleon approach the integrand of (41) must be multiplied by a model dependent correction factor

$$h_{corr,k} = \frac{V_{f\mu}^{St}(type, \mathbf{k}_{\mu})}{V_{f\mu}^{St,W}(type)}, \quad (45)$$

of the  $k$ -th target particle, where  $V_{f\mu}^{St}(type, \mathbf{k}_{\mu})$  is the nuclear matrix element calculated in an other model, without the long wavelength approximation and beyond the single nucleon approach.

We will mainly treat proton (and deuteron) capture processes in which the interaction of proton (deuteron) takes place with more than one nucleon. In the Weisskopf approximation the sum of the matrix elements of proton-proton and neutron-neutron interactions can be neglected due to the presence of exchange terms, so in the case of proton capture the matrix element of the strong interaction must be multiplied by the number of interacting neutrons in the nucleus. In our case it means the neutron number of particle 3  $\widehat{N}_3 = A_3 - z_3$  ( $h_{corr,3} = \widehat{N}_3$ ). In the case of deuteron capture reactions and also in the Weisskopf approximation the matrix-element must be multiplied by  $A_3$  ( $h_{corr,3} = A_3$ ).

### A. Rates of electron assisted $p$ or $d$ capture processes

First the case  $type = \alpha$  is treated. Neglecting the initial kinetic energies and  $E_{2\mu}$  in the denominator of (12),  $E_\mu - E_i = E_{1f} + E_{2\mu} \simeq E_{1f}$  that will have a value  $E_{1f} \simeq \Delta$  because of the energy Dirac delta. Using the  $\sum_\mu \rightarrow \int [V/(2\pi)^3] d\mathbf{k}_{2\mu}$  and the  $\sum_f \rightarrow \int [V/(2\pi)^3] d\mathbf{k}_4 \times \int [V/(2\pi)^3] d\mathbf{k}_{1f}$  substitutions, the identities  $[(2\pi)^3 \delta(\mathbf{k}_4 + \mathbf{k}_{1f})]^2 = (2\pi)^3 \delta(\mathbf{k}_4 + \mathbf{k}_{1f}) (2\pi)^3 \delta(\mathbf{0})$  and  $(2\pi)^3 \delta(\mathbf{0}) = V$ , and the  $k_{1f} dk_{1f} = E_{1f} dE_{1f} / (\hbar c)^2$  relations, furthermore carrying out integrations one can obtain

$$W_{fi}^{(2)}(\alpha) = K_0(\alpha) G_S(k_{1i}, 2k_{1f}) \frac{F_{23}(k_{1f})}{V^2 \Delta^4} h_{corr,3}^2. \quad (46)$$

Here  $G_S(k_{1i}, 2k_{1f}) = g_S^2$  [see (10)], remember that  $F_{jk}(k) = f_{jk}^2(k)$  [see (8)] and  $K_0(\alpha)$  is defined in (20). In the case  $E_{1f} \gg m_e c^2$

$$k_{1f} = \Delta / (\hbar c). \quad (47)$$

If one is interested in the total rate  $W_{tot}^{(2)}(\alpha)$  of a sample then  $W_{fi}^{(2)}$  must be multiplied by the numbers of initial particles,  $N_1$ ,  $N_2$  and  $N_3$  corresponding to electrons, and particles 2 and 3, respectively. The quantity  $N_1/V = g_e/v_c$ , where  $v_c$  is the volume of the elementary cell of the solid metal and  $g_e$  is the number of the valence electron states corresponding to one unit cell, e.g.  $g_e = 10$  and  $v_c = d^3/4$  in the cases of  $Ni$  and  $Pd$  will be discussed later. We introduce  $N_3/V = n_3$ , the number density of particles 3. It is reasonable to take for the number density  $n_3$  of the target  $n_3 = 2u/v_c = u8/d^3$  in the case of  $fcc$  metals (such as  $Pd$  and  $Ni$ ) with  $u$  denoting the deuteron (or proton) over metal number densities. The result is

$$W_{tot}^{(2)}(\alpha) = K_{tot}(\alpha) \langle G_S(k_{1i}, 2k_{1f}) \rangle_{av} \frac{F_{23}(\frac{\Delta}{\hbar c})}{\Delta^4} h_{corr,3}^2 u N_2 \quad (48)$$

where  $K_{tot}(\alpha)$  is determined by (19) and the average is made by means of Fermi-Dirac distribution.

### B. Rates of electron assisted processes with two outgoing heavy particles

In the case of  $type = +\beta$  or  $n\beta$  (for the process (b) of FIG.1) the calculation is modified as follows. Let the wave vectors of particles 5 and 6 be  $\mathbf{k}_5$  and  $\mathbf{k}_6$ . This time, the  $\sum_f \rightarrow \int [V/(2\pi)^3] d\mathbf{k}_5 \times \int [V/(2\pi)^3] d\mathbf{k}_6 \times \int [V/(2\pi)^3] d\mathbf{k}_{1f}$  correspondence and the identities

$[\delta(\mathbf{k}_{1f} + \mathbf{k}_5 + \mathbf{k}_6)]^2 = \delta(\mathbf{k}_{1f} + \mathbf{k}_5 + \mathbf{k}_6) \delta(\mathbf{0})$  and  $(2\pi)^3 \delta(\mathbf{0}) = V$  are used. The integration over  $\mathbf{k}_{1f}$  results  $\mathbf{k}_{1f} = -\mathbf{k}_5 - \mathbf{k}_6$ . Using again the  $N_3/V = n_3 = 2u/v_c = u8/d^3$  relation,

$$W_{tot}^{(2)}(\beta) = K_{tot}(\alpha) \frac{\langle G_S(k_{1i}, 2k_{1f}) \rangle_{av}}{6\pi^2 (1 - \frac{2}{e})^2} \left( \frac{R}{\hbar c} \right)^3 \frac{J}{\Delta} \hbar_{corr,3}^2 u N_2 \quad (49)$$

with

$$J = \int \int \frac{\delta(\varepsilon)}{|\kappa_5 + \kappa_6|^4 \varepsilon_{1f}^2(\kappa_5 + \kappa_6)} \times \quad (50)$$

$$\times F_{23}\left(\frac{\Delta}{\hbar c} |\kappa_5 + \kappa_6|\right) F_{56}\left(\frac{\Delta}{\hbar c} |\kappa_5 - \kappa_6|\right) d\kappa_5 d\kappa_6,$$

where

$$\delta(\varepsilon) = \delta[\varepsilon_5(\kappa_5) + \varepsilon_6(\kappa_6) + \varepsilon_{1f}(\kappa_5 + \kappa_6) - \delta_e - 1]. \quad (51)$$

Here  $\kappa_j = \hbar c \mathbf{k}_j / \Delta$  and  $\varepsilon_j = E_j / \Delta = [\kappa_j^2 / (2m_j)] \Delta$  are dimensionless quantities with  $E_j$  as the kinetic energy of particle  $j$  ( $j = 5$  or  $6$ ),  $\delta_e = m_e c^2 / \Delta$ ,  $\varepsilon_{1f} = E_{1f} / \Delta = \sqrt{|\kappa_5 + \kappa_6|^2 + \delta_e^2} - \delta_e$  and the suffixes 23 of  $F_{23}$  and 56 of  $F_{56}$  refer to particles 2, 3 and 5, 6, respectively. If one of the particles 5 and 6 is a neutron then

$$F_{56} = 1. \quad (52)$$

It is useful to introduce the following new variables in (50):

$$\mathbf{a} = \kappa_5 + \kappa_6 \quad (53)$$

and

$$\mathbf{b} = \kappa_5 - \kappa_6. \quad (54)$$

Thus, if both particles 5 and 6 are positively charged then  $J = J_{+\beta}$ ,

$$J_{+\beta} = A_{23} A_{56} J_2 \quad (55)$$

where

$$A_{23} = 2\pi z_2 z_3 \alpha_f \mu_{23} c^2 / \Delta, \quad (56)$$

$$A_{56} = 2\pi z_5 z_6 \alpha_f \mu_{56} c^2 / \Delta \quad (57)$$

and

$$J_2 = \int \int \frac{\delta[\varepsilon(\mathbf{a}, \mathbf{b})]}{|\mathbf{a}|^5 |\mathbf{b}| \varepsilon_{1f}^2(\mathbf{a})} \times \quad (58)$$

$$\times \exp\left(-\frac{A_{23}}{|\mathbf{a}|}\right) \exp\left(-\frac{A_{56}}{|\mathbf{b}|}\right) d\mathbf{a} d\mathbf{b}.$$

If one of the particles 5 and 6 is a neutron then  $J = J_{n\beta}$ ,

$$J_{n\beta} = A_{23}J_3 \quad (59)$$

with

$$J_3 = \int \int \frac{\delta[\varepsilon(\mathbf{a}, \mathbf{b})]}{|\mathbf{a}|^5 \varepsilon_{1f}^2(\mathbf{a})} \exp\left(-\frac{A_{23}}{|\mathbf{a}|}\right) d\mathbf{a}d\mathbf{b}. \quad (60)$$

Here and above

$$\delta[\varepsilon(\mathbf{a}, \mathbf{b})] = \delta\left(\frac{\Delta}{8\mu_{56}c^2}b^2 + \sqrt{a^2 + \delta_e^2} - 1 - \delta_e\right). \quad (61)$$

In obtaining (61) the  $\Delta a^2/(8\mu_{56}c^2)$  and  $\Delta\left(\frac{1}{4m_5c^2} - \frac{1}{4m_6c^2}\right)\mathbf{a} \cdot \mathbf{b}$  terms in the argument can be neglected since  $\Delta/(\mu_{56}c^2) \ll 1$ ,  $a \lesssim 1$ , and the dominant range in the integrals is where  $b \gg a$ . Using the  $\delta[g(x)] = \delta(x - x_1)/g'(x_1)$  identity, where  $x_1$  is the root of the equation  $g(x) = 0$ , the integrals  $J_2$  and  $J_3$  can be written as

$$J_2 = \beta_2 J_{22} \quad (62)$$

with

$$\beta_2 = 64\pi^2 \mu_{56}c^2 / \Delta, \quad (63)$$

$$J_{22} = \int_0^{a_{\max}} \frac{\exp\left(-\frac{A_{23}}{a}\right) \exp\left(-\frac{A_{56}\sqrt{\Delta/(8\mu_{56}c^2)}}{\sqrt{1+\delta_e-\sqrt{a^2+\delta_e^2}}}\right) da}{a^3 \left(\sqrt{a^2 + \delta_e^2} - \delta_e\right)^2} \quad (64)$$

and

$$J_3 = \beta_3 J_{32} \quad (65)$$

with

$$\beta_3 = 512\pi^2 (\mu_{56}c^2 / \Delta)^2, \quad (66)$$

$$J_{32} = \int_0^{a_{\max}} \frac{\exp\left(-\frac{A_{23}}{a}\right) \left(1 + \delta_e - \sqrt{a^2 + \delta_e^2}\right) da}{a^3 \left(\sqrt{a^2 + \delta_e^2} - \delta_e\right)^2} \quad (67)$$

where  $a_{\max} = \sqrt{1 + 2\delta_e}$ . Using all the above results, the total rate  $W_{tot}^{(2)}(+\beta)$  of the process having two charged, heavy products reads

$$W_{tot}^{(2)}(+\beta) = K_{tot}(\alpha) K_2 K_+ \langle G_S(k_{1i}, 2k_{1f}) \rangle_{av} \frac{J_{22}}{\Delta^4} h_{corr,3}^2 u N_2, \quad (68)$$

where

$$K_2 = z_2 z_3 \frac{512}{3 \left(1 - \frac{2}{e}\right)^2} \pi \left(\frac{R}{\hbar c}\right)^3 \alpha_f \mu_{23} c^2 (\mu_{56}c^2)^2 \quad (69)$$

and

$$K_+ = \frac{\pi}{4} \alpha_f z_5 z_6. \quad (70)$$

The total rate  $W_{tot}^{(2)}(n\beta)$  of the process, in which one of the two heavy products is a neutron, reads

$$W_{tot}^{(2)}(n\beta) = K_{tot}(\alpha) K_2 \langle G_S(k_{1i}, 2k_{1f}) \rangle_{av} \frac{J_{32}}{\Delta^4} \hbar^2_{corr,3} u N_2. \quad (71)$$

In order to compare the total rates (48), (68) and (71) of the processes of different type, the quantities  $F_{23}(\frac{\Delta}{\hbar c})$ ,  $K_2 K_+ J_{22}$  and  $K_2 J_{32}$  have to be compared. The effect of factor  $\langle G_S(k_{1i}, 2k_{1f}) \rangle_{av}$  is not essential in the electron assisted processes.

## VIII. APPENDIX II. - RATE CALCULATION OF HEAVY PARTICLE ASSISTED NUCLEAR PROCESSES

In the case of heavy particle assisted nuclear processes the quantity  $T_{if}^{(2)}$  modifies as

$$T_{if}^{(2)}(type) = B(type) z_1 z_2 \delta(\mathbf{k}_{1f} + \mathbf{K}_{(type)}) \times \int \frac{g_S f_{23}(k_{2\mu}) \tilde{\psi}_{1i}(\mathbf{k}_{2i} - \mathbf{k}_{1f} - \mathbf{k}_{2\mu})}{(E_\mu - E_i) |\mathbf{k}_{2i} - \mathbf{k}_{2\mu}|^2 + \lambda^2} d\mathbf{k}_{2\mu} \quad (72)$$

where

$$E_\mu = \frac{\hbar^2}{2m_1} k_{1\mu}^2 + \frac{\hbar^2}{2m_2} k_{2\mu}^2, \quad (73)$$

$$B(\alpha) = B_0 \frac{\sqrt{3\pi R}}{V^2} \quad (74)$$

with

$$B_0 = -8 (2\pi)^4 \alpha_f \left(1 - \frac{2}{e}\right) \left(\frac{f^2}{\hbar c}\right) (\hbar c)^2, \quad (75)$$

$$B(n\beta) = \frac{B(\alpha)}{\left(1 - \frac{2}{e}\right)} \sqrt{\frac{4\pi R^3}{3V}} \quad (76)$$

and

$$B(+\beta) = B(n\beta) f_{56}. \quad (77)$$

Since  $\beta_1 \ll |\mathbf{K}| \approx |\mathbf{k}_{1f}|$ , the Fourier transform of (24)

$$\tilde{\psi}_{1i}(\mathbf{K}) = \frac{2^{3/2} \pi^{3/4}}{\beta_1^{3/2}} e^{-\frac{\mathbf{K}^2}{2\beta_1^2}} \quad (78)$$

allows the approximation

$$\tilde{\psi}_{1i}(\mathbf{K}) = 8\pi^{9/4}\beta_1^{3/2}\delta(\mathbf{K}) \quad (79)$$

in (72). As a result the integral over  $\mathbf{k}_{2\mu}$  in (72) can be carried out, while  $|\mathbf{k}_{2i}|$  and  $\lambda$  can be neglected since  $|\mathbf{k}_{2i}| \ll |\mathbf{k}_{1f}|$  and  $\lambda \ll |\mathbf{k}_{1f}|$ . Neglecting  $E_i$  in the denominator,  $E_\mu - E_i = \hbar^2 k_{1f}^2 / (2\mu_{12})$ . The result for  $T_{if}^{(2)}(type)$  obtained in this way is

$$T_{if}^{(2)}(type) = B(type)z_1z_2\delta(\mathbf{k}_{1f} + \mathbf{K}_{(type)}) \times \quad (80)$$

$$g_S(k_{2i}, 2k_{1f})\mu_{12}\frac{f_{23}(k_{1f})}{\hbar^2 k_{1f}^2} \frac{16\pi^{9/4}\beta_1^{3/2}}{k_{1f}^2}.$$

### A. Rates of heavy particle assisted nuclear capture processes

First the process of  $type = \alpha$  is dealt with. Substituting (80) into (11) the  $\sum_f \rightarrow \int [V/(2\pi)^3] d\mathbf{k}_4 \times \int [V/(2\pi)^3] d\mathbf{k}_{1f}$  correspondence, the identities  $[\delta(\mathbf{k}_{1f} + \mathbf{k}_4)]^2 = \delta(\mathbf{k}_{1f} + \mathbf{k}_4)\delta(\mathbf{0})$  and  $(2\pi)^3\delta(\mathbf{0}) = V$  are used. When integrating over  $\mathbf{k}_4$ , the  $\delta(\mathbf{k}_{1f} + \mathbf{k}_4)$  leads to the  $\mathbf{k}_4 = -\mathbf{k}_{1f}$  replacement. Therefore  $E_f = E_{1f}(k_{1f}) + E_4(k_{1f}) = \hbar^2 k_{1f}^2 / (2\mu_{14})$  in the energy Dirac delta. Applying the  $k_{1f}dk_{1f} = \mu_{14}dE_f/\hbar^2$  and  $k_{1f} = \sqrt{2\mu_{14}E_f}/\hbar$  relations the integral over  $k_{1f}$  is converted to an integral over  $E_f$  and it is carried out with the aid of the energy Dirac delta. Thus the transition rate of a heavy particle assisted reaction of  $type = \alpha$

$$W_{fi}^{(2),h}(\alpha) = \frac{K_0^h(\alpha)\beta_1^3}{\sqrt{m_0c^2}}G_S(k_{2i}, 2k_{1f})h_{corr,3}^2 \times \quad (81)$$

$$\times \left(\frac{a_{12}}{a_{14}}\right)^2 \frac{(z_1z_2)^2}{\sqrt{a_{14}}} \frac{F_{23}^h(\Delta)}{\Delta^{7/2}} \frac{1}{V}.$$

For  $K_0^h(\alpha)$  see (26), the quantity  $F_{23}^h(\Delta)$  is given by (27), the parameter  $\eta_{23}^h(\Delta)$  is determined by (28),  $m_0c^2 = 931.494 [MeV]$  is the atomic mass unit, and  $a_{jk}$  is the reduced mass number of particles  $j$  and  $k$  of mass numbers  $A_j$  and  $A_k$  given by (29).

The total rate  $W_{tot}^{(2),h}(\alpha)$  can be obtained multiplying  $W_{fi}^{(2),h}(\alpha)$  by  $N_2$  and  $N_{13}$ .  $N_2$  is the number of particles 2 and  $N_{13}$  is the number of possible pairs of particles 1 and 3. Introducing  $n_{13} = N_{13}/V$ , the number density of particles 1 and 3, we take again  $n_{13} = 2u/v_c = 8u/d^3$

valid in the case of *fcc* metals.

$$W_{tot}^{(2),h}(\alpha) = \frac{8K_0^h(\alpha)\beta_1^3}{d^3\sqrt{m_0c^2}} G_S(k_{2i}, 2k_{1f}) h_{corr,3}^2 \times \left(\frac{a_{12}}{a_{14}}\right)^2 \frac{(z_1z_2)^2}{\sqrt{a_{14}}} \frac{F_{23}^h(\Delta)}{\Delta^{7/2}} uN_2. \quad (82)$$

The parameter  $\beta_1$  in the case of a localized deuteron is  $\beta_1^d = \sqrt{m_d\omega/\hbar}$ , where  $m_d$  is the deuteron rest mass. In *Pd*, that is one of the possible materials which can absorb hydrogen isotopes well, the energy of the ground state of the oscillator  $E_0 = \frac{3}{2}\hbar\omega = 72$  [meV] [14] leading to  $\hbar\omega = 48$  [meV], that corresponds to the energy of an optical phonon, and  $\beta_1^d = 4.81 \times 10^8$  [cm<sup>-1</sup>]. This value alters as  $\beta_1^p = 4.04 \times 10^8$  [cm<sup>-1</sup>] for protons. With these numbers the characteristic constants, which appear in  $W_{tot}^{(2),h}(\alpha)$  (82), are

$$\frac{8K_0^h(\alpha)(\beta_1^d)^3}{d^3\sqrt{m_0c^2}} = 190 \text{ [MeV}^{7/2}\text{s}^{-1}] \quad (83)$$

and

$$\frac{8K_0^h(\alpha)(\beta_1^p)^3}{d^3\sqrt{m_0c^2}} = 113 \text{ [MeV}^{7/2}\text{s}^{-1}] \quad (84)$$

in the case of deuterized and protonated *Pd*, and in the cases when particle 1 is a deuteron or a proton, respectively.

## B. Rates of heavy particle assisted processes with two outgoing heavy particles

In the case of *type* =  $+\beta$  or  $n\beta$  (for the process (b) of FIG. 2) the calculation is modified as follows. Now the  $\sum_f \rightarrow \int [V/(2\pi)^3] d\mathbf{k}_5 \times \int [V/(2\pi)^3] d\mathbf{k}_6 \times \int [V/(2\pi)^3] d\mathbf{k}_{1f}$  correspondence and the identities  $[\delta(\mathbf{k}_{1f} + \mathbf{k}_5 + \mathbf{k}_6)]^2 = \delta(\mathbf{k}_{1f} + \mathbf{k}_5 + \mathbf{k}_6) \delta(\mathbf{0})$  and  $(2\pi)^3 \delta(\mathbf{0}) = V$  are used and the integration over  $\mathbf{k}_{1f}$  leads to the  $\mathbf{k}_{1f} = -\mathbf{k}_5 - \mathbf{k}_6$  replacement. In the argument of the Dirac delta  $E_f = E_{1f}(|\mathbf{k}_5 + \mathbf{k}_6|) + E_5(k_5) + E_6(k_6)$ , which can be rewritten as

$$E_f = \frac{\hbar^2 k_5^2}{2\mu_{15}} + \frac{\hbar^2 k_6^2}{2\mu_{16}} + \frac{\hbar^2}{m_1} k_5 k_6 \zeta, \quad (85)$$

where  $\zeta = \cos \Theta_{56}$  with  $\Theta_{56}$  the angle of vectors  $\mathbf{k}_5$  and  $\mathbf{k}_6$ . Now new variables

$$x_j = \sqrt{\frac{\hbar^2}{2m_1\Delta}} k_j, \quad j = 5, 6 \quad (86)$$

are introduced. With these variables

$$E_f = \left( \frac{m_1}{\mu_{15}} x_5^2 + \frac{m_1}{\mu_{16}} x_6^2 + 2x_5 x_6 \zeta \right) \Delta \quad (87)$$

and

$$k_{1f}^2 = \frac{2m_1\Delta}{\hbar^2} (x_5^2 + x_6^2 + 2x_5x_6\zeta). \quad (88)$$

Applying the  $\delta[\chi(\zeta)] = |\chi'(\zeta_1)|^{-1} \delta(\zeta - \zeta_1)$  identity, where  $\zeta_1$  is the root of equation  $\chi(\zeta) = 0$ , the integration over  $\zeta$  can be carried out. In our case

$$\zeta_1 = \frac{1 - \left(\frac{m_1}{\mu_{15}}x_5^2 + \frac{m_1}{\mu_{16}}x_6^2\right)}{2x_5x_6}. \quad (89)$$

Introducing the notation

$$g_{(\pm)}(x_5, x_6) = x_5^2 + x_6^2 \pm 2x_5x_6\zeta_1 \quad (90)$$

and after some algebra the transition rate

$$\begin{aligned} W_{fi}^{(2),h}(\text{type}) &= \frac{K_0(\beta)\beta_1^3}{m_1c^2} G_S(k_{2i}, 2k_{1f}) h_{corr,3}^2 \times \\ &\times (z_1z_2)^2 \left(\frac{\mu_{12}c^2}{\Delta}\right)^2 J_{\text{type}} \frac{1}{V}, \end{aligned} \quad (91)$$

where

$$K_0(\beta) = 2^{16}\pi^{11/2}\alpha_f^2 \left(\frac{f^2}{\hbar c}\right)^2 R^4 \hbar c^2. \quad (92)$$

For  $\text{type} = +\beta$  the integral  $J_{\text{type}}$  is

$$J_{+\beta} = \int_0^{x_{6,up}} \int_0^{x_{5,up}} \frac{\Psi(\xi_{23})\Psi(\xi_{56}^-)}{g_+^4(x_5, x_6)} x_5 dx_5 x_6 dx_6 \quad (93)$$

and for  $\text{type} = n\beta$  the integral  $J_{\text{type}}$  is

$$J_{n\beta} = \int_0^{x_{6,up}} \int_0^{x_{5,up}} \frac{\Psi(\xi_{23})}{g_+^4(x_5, x_6)} x_5 dx_5 x_6 dx_6. \quad (94)$$

Here the following notation is used:

$$\Psi(\xi) = \frac{\xi}{\exp(\xi) - 1}, \quad (95)$$

and the variables are

$$\xi_{23} = 2\pi z_2 z_3 \alpha_f \frac{\mu_{23}c}{\sqrt{2m_1\Delta}} g_{(+)}^{-1/2}(x_5, x_6) \quad (96)$$

and

$$\xi_{56}^- = 2\pi z_5 z_6 \alpha_f \frac{\mu_{56}c}{\sqrt{2m_1\Delta}} g_{(-)}^{-1/2}(x_5, x_6) \quad (97)$$

with

$$g_{(\pm)}(x_5, x_6) = x_5^2 \left(1 \mp \frac{m_1}{\mu_{15}}\right) + x_6^2 \left(1 \mp \frac{m_1}{\mu_{16}}\right) \pm 1. \quad (98)$$

The upper limits  $x_{5,up}$  and  $x_{6,up}$  are determined by the condition  $|\zeta_1| \leq 1$ .

Now the total rates  $W_{tot}^{(2),h}(type)$  of reactions  $type = +\beta$  or  $n\beta$  can also be obtained multiplying  $W_{fi}^{(2),h}(type)$  by  $N_2$  and  $N_{13}$ , and using  $n_{13} = N_{13}/V = 8u/d^3$

$$W_{tot}^{(2),h}(type) = \frac{8K_0(\beta)\beta_1^3}{d^3m_1c^2} G_S(k_{2i}, 2k_{1f}) h_{corr,3}^2 \times \quad (99)$$

$$\times (z_1 z_2)^2 \left( \frac{\mu_{12}c^2}{\Delta} \right)^2 J_{type} u N_2.$$

In the rates  $W_{tot}^{(2),h}(\alpha)$  (82) and  $W_{tot}^{(2),h}(type)$  (99) the function  $G_S(k_{2i}, 2k_{1f}) = F_{12}(k_{2i})/F_{12}(2k_{1f})$  plays important role since  $F_{12}(k_{2i}(E_i))$  has strong energy dependence. If e.g. both particles 1 and 2 are protons then  $2\pi\eta_{12}(E_{2i}) = 0.700 \times E_{2i}^{-1/2}$  resulting  $F_{12}(k_{2i}(E_{2i})) = 6.4 \times 10^{-3}$  at  $E_{2i} = 0.01$  [MeV].

It is also an important aspect of the mechanism that if particle 1 is a proton then in each proton assisted nuclear capture process an energetic proton (of energy of about a few MeV) is created too. This proton in its decelerating processes can create (secondary) free protons in the crystal, if localized protons are also present. The secondary free protons may take part in further proton assisted nuclear capture processes. Thus in this process the secondary protons can play a role similar to that played by the secondary neutrons in the case of nuclear fission.

## IX. APPENDIX III. - PROTON ASSISTED PROTON CAPTURE PROCESSES

Here some cases of the proton assisted proton captures



are investigated. Particles 1 and 2 are protons. Particles 3 and 4 have mass numbers  $A$  and  $A + 1$ , respectively (see FIG. 2). As a first example we take protonated  $Ni$ . In this case the possible processes are



Most of the daughter nuclei  ${}^{A+1}_{29} Cu$  decay by the



electron capture reaction. TABLE I. contains the relevant data for reactions (101) and (102).

$A$	58	59	60	61	62	64
$\Delta(\text{MeV})$	3.417	4.479	4.801	5.867	6.122	7.452
$r_A$	0.68077	$(2.4 \times 10^{12}\text{s})$	0.26223	0.0114	0.03634	0.00926
$2\pi\eta_{23}^h$	15.04	13.14	12.69	11.48	11.24	10.18
$s_A(\Delta) \times 10^8(\text{MeV}^{-7/2})$	4.08	–	4.23	0.276	0.945	0.316
$\tau(\text{s})$	81.5	1422	$1.20 \times 10^4$	584.4	stable	stable
$Q_{EC}(\text{MeV})$	4.789	6.127	2.237	3.948	-	-

TABLE I: Numerical data of the  ${}^A_{28}\text{Ni} + p \rightarrow {}^{A+1}_{29}\text{Cu} + \Delta$  and  ${}^{A+1}_{29}\text{Cu} + e \rightarrow {}^A_{28}\text{Ni} + Q_{EC}$  reactions. Computed values of  $2\pi\eta_{23}^h$  and  $s_A(\Delta)$  are given (for their formula see the text).  $A$  is the mass number,  $r_A$  is the relative natural abundance,  $\tau$  is the half life of the  ${}^{A+1}_{29}\text{Cu}$  isotope produced. It decays by electron capture of reaction energy  $Q_{EC}$ . In the row  $r_A$  at  $A = 59$  the  $2.4 \times 10^{12}$  s stands for the life time of the unstable  ${}^{59}_{28}\text{Ni}$ .

An other interesting family of the heavy particle (proton) assisted proton capture is



that is mainly followed by the



reaction. (The relevant data can be found in TABLE II. The nuclear data of TABLES I. and II. are taken from [15].)

For the processes (101) and (103)  $a_{12} = 1/2$  and  $a_{14} = a_{23} = 1$  is a good approximation in the case of protonated  $\text{Ni}$  and  $\text{Pd}$ . First we take a protonated  $\text{Ni}$  ( $z_3 = 28$ ). The rate  $W_{tot}^{(2),h}(\alpha, A)$  belonging to process (101) is

$$W_{tot}^{(2),h}(\alpha, A) = G_S \frac{2K_0^h(\alpha) (\beta_1^p)^3}{d^3 \sqrt{m_0 c^2}} (A - z_3)^2 s_A(\Delta) u N_2. \quad (105)$$

Here  $K_0^h(\alpha)$  is given by (26),  $d$  is the lattice parameter,  $m_0 c^2 = 931.494$  [MeV] is the atomic mass unit, and for  $\beta_1^p$  see Appendix II. A under (82). These all result

$A$	102	104	105	106	108	110
$\Delta(\text{MeV})$	4.155	4.966	5.814	5.789	6.487	7.156
$r_A$	0.0102	0.1114	0.2233	0.2733	0.2646	0.1172
$2\pi\eta_{23}^h$	22.41	20.49	18.94	18.98	17.93	17.07
$s_A(\Delta) \times 10^{11}(\text{MeV}^{-7/2})$	0.0285	1.06	5.31	6.35	11.1	7.80
$\tau(s)$	4002	$3.56 \times 10^6$	1438	stable	stable	$6.44 \times 10^5$
$Q_{EC}(\text{MeV})$	2.688	1.346	2.965	-	-	no EC

TABLE II: Numerical data of the  ${}^A_{46}\text{Pd} + p \rightarrow {}^{A+1}_{47}\text{Ag} + \Delta$  and  ${}^{A+1}_{47}\text{Ag} + e \rightarrow {}^{A+1}_{46}\text{Pd} + Q_{EC}$  reactions. Computed values of  $2\pi\eta_{23}^h$  and  $s_A(\Delta)$  are given (for their formula see the text).  $A$  is the mass number,  $r_A$  is the relative natural abundance,  $\tau$  is the half life of the  ${}^{A+1}_{47}\text{Ag}$  isotope produced. It decays by electron capture of reaction energy  $Q_{EC}$ .

$2K_0^h(\alpha)(\beta_1^p)^3 d^{-3}(m_0c^2)^{-1/2} = 28.3 [MeV^{7/2}s^{-1}]$ .  $N_2$  is the actual number of free protons in the sample,

$$s_A(\Delta) = r_A \frac{F_{23}^h(\Delta)}{\Delta^{7/2}}, \quad (106)$$

where  $r_A$  is the relative natural abundance and

$$2\pi\eta_{23}^h = \frac{27.8 [MeV^{1/2}]}{\sqrt{\Delta}} \quad (107)$$

with  $z_3 = 28$  in (28). The formula (105) can be also used in the case of processes (103) with  $s_A$  and  $\Delta$  data of TABLE II. and using

$$2\pi\eta_{23}^h = \frac{45.7 [MeV^{1/2}]}{\sqrt{\Delta}}, \quad (108)$$

that comes from (28) with  $z_3 = 46$  (corresponding to  $Pd$ ).

The processes (101) and (103) may produce stable  ${}^{62}_{29}\text{Cu}$ ,  ${}^{64}_{29}\text{Cu}$  and  ${}^{106}_{47}\text{Ag}$ ,  ${}^{108}_{47}\text{Ag}$  isotopes, respectively, whose heavy particle assisted proton capture reaction may give rise to a chain

of nuclear transmutations.

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